A Multivariate EWMA Approach to Monitor Process Dispersion

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Since the development of control charts by Shewhart in 1926, it has become a common practice to rely on statistical control schemes to monitor a process. The underlying idea is, if one wishes to produce high-quality products, one should control the process and not completely inspect the finished product.

Various charts are being used by practitioners to monitor different processes. $X-$ bar and S charts, for instance, are often used in the univariate case to monitor the center and the spread of a process, respectively. There are many situations in industry where the simultaneous control of two or more quality characteristics is necessary; this has led to the development of multivariate quality control procedures.

During the past decades, a number of multivariate procedures have been developped to monitor a process mean vector; included among those, the Chi-square control chart by Alt (1974), the Multivariate CUSUM by Croisier et al., and the Multivariate EWMA by Lowry et al. Very little is published on multivariate control charts for monitoring the covariance matrix. Alt (1985), Alt and Bedewi (1986), and Alt and Smith (1988) propose three control charts. The first one is based on the likelihood ratio test for testing whether the covariance matrix $\mathbf{\Sigma}$ is equal to a given covariance matrix $\mathbf{\Sigma}_0$. The second and third charts are based on the generalized sample variance, denoted by $|\mathbf{S}|$. All three of these procedures are Shewhart-type charts.

A major disadvantage of any Shewhart control chart is that it only uses the information about the process contained in the last plotted point and that ignores any information given by any sequence of past points. Thus, those charts tend to be more sensitive to large shifts. A good alternative to the Shewart control chart when interested in detecting small shifts is the Exponential Weighted Moving Average (EWMA) introduced by Roberts (1959).

Most applications of the EWMA for process monitoring have been concentrated on the problem of detecting shifts in the mean level of a process. Until Crowder and Hamilton (1992), little attention has been given to the use of EWMA for monitoring process variability, which can have a major impact on product quality.

The main purpose of this research is to develop an effective EWMA pro-

cedure for monitoring the dispersion of a multivariate process. Properties of the EWMA chart based on the log transformation of the generalized sample variance are presented. Average run lengths (ARL) tables and plots are generated to facilitate the design of an optimal EWMA control chart. It is shown that the optimal EWMA procedure performs better than the Shewhart one in terms of its ability to quickly detect small shifts in process variability.