Integration Trick

If you're having trouble "thinking sideways" feel free to use the following trick:

Suppose you want to integrate

$$\int_0^1 \int_0^{y^2} f(x,y)\,dx\,dy$$

This involves an inner integral where x is integrated from $x=0$ to $x=y^2$. But you may have trouble visualizing/graphing the $x=y^2$ part.

So remember, the variables of integration are "dummy variables", that is, you could replace them with any other variable and get the same thing. So you could replace x with s and y with t if you wanted.

You can even replace x with y and y with x! Just remember, you must change them <u>everywhere</u> they occur, including the integrand.

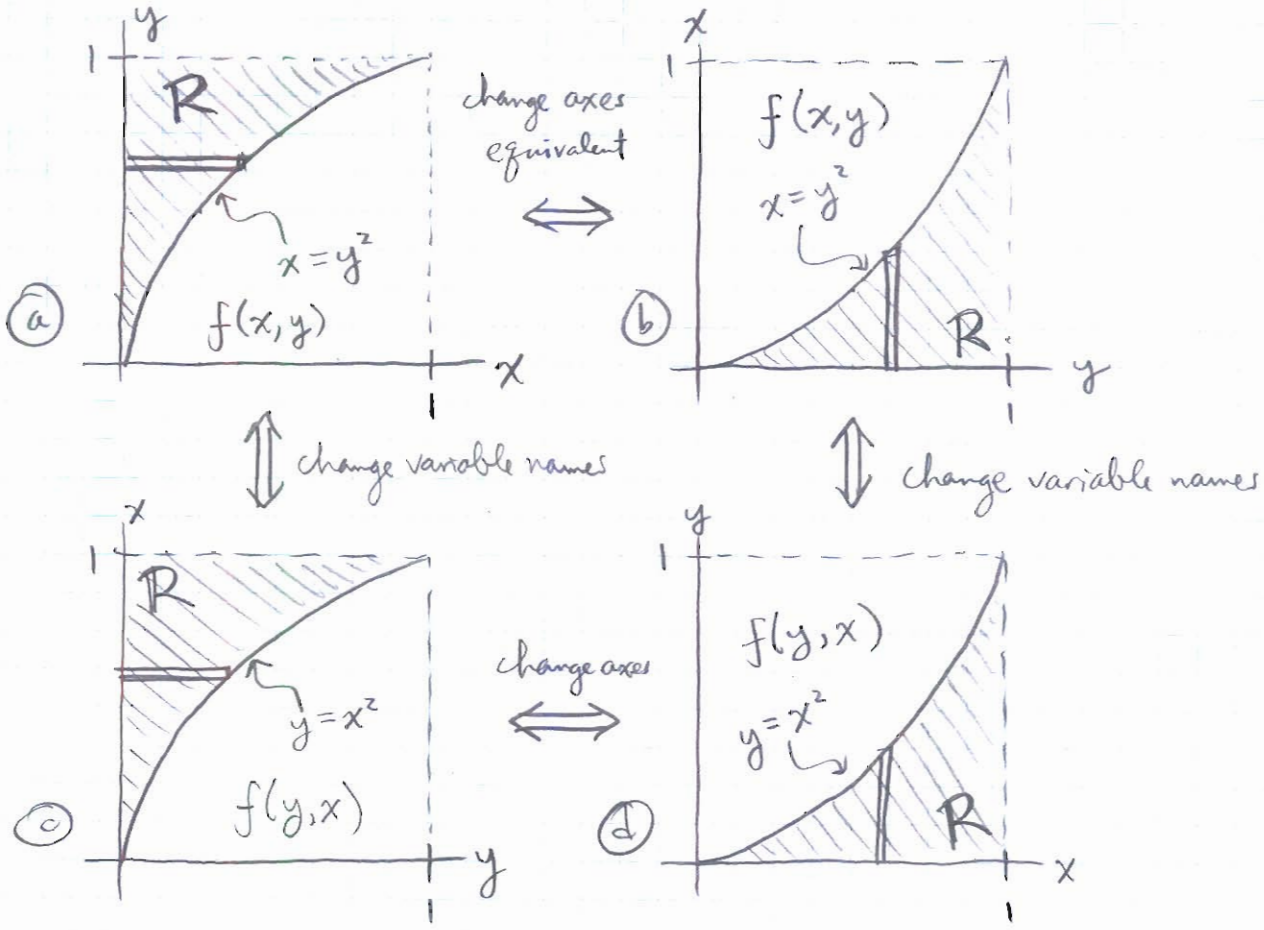
Thus, we can rewrite the integral above as:

$$\int_0^1 \int_0^{x^2} f(y,x)\,dy\,dx \qquad \text{(Note the new integrand)}$$

And (hopefully) you know what $y=x^2$ looks like!

*<u>NOTE</u>: I have <u>not</u> changed the order of integration, just the <u>names</u> of the dummy variables...

This is really just the same thing as plotting the variables in the yx-plane rather than the xy-plane.



All four of these are equivalent, and they all represent the same order of integration.

Probably only (d) is useful... But anyway, you want to get used to doing (a) directly to save time, but until you "get it", this may help.