MATH 241 **FINAL EXAM** **May 12, 2007**

Instructions: Number the answer sheets from 1 to 9. Fill out all the information at the top of each sheet. Answer problem n on page n, $n = 1, \cdots, 9$. Do not answer one question on more than one sheet. If you need more space use the back of the correct sheet. Please write out and sign the **Honor Pledge** on page 1 only.

SHOW ALL WORK

The Use of Calculators Is Not Permitted On This Exam

1. (20 points)
(a) Find equations for the line L through $P_0 = (6, 3, 5)$ perpendicular to the plane $\mathcal{P} : 2x + y + 3z = 2$. Give both a symmetric and a parametric form.
(b) Find the point of intersection of L and $\mathcal{P}$.

2. (20 points) Let

$$\mathbf{r}(t) = 2t^{3/2}\mathbf{i} + 4\cos t\mathbf{j} + 4\sin t\mathbf{k} \text{ for } 0 \leq t \leq 1.$$

(a) Find $\mathbf{T}$ the unit tangent vector.
(b) Find the length L of the curve.

3. (25 points) Let $f(x, y) = x^2 - y^2 + 2xy + 4x - 8y + 3$.
(a) Find the directional derivative of f at the point $P = (1, 1)$ in the direction of $\mathbf{a} = \mathbf{i}+2\mathbf{j}$.
(b) Find all critical points of f.
(c) Show that f has no absolute maximum or absolute minimum.

4. (20 points) Let P_0 be the point and $\mathcal{P}$ be the plane of Problem 1. Use the method of Lagrange multipliers to find the point on $\mathcal{P}$ which is closest to P_0. Compare your answer to the answer of Problem 1(b). (Note: it is easier to minimize the square of the distance from P_0 to points on the plane.) To get credit for this problem you must use Lagrange multipliers.

5. (25 points) Find the center of gravity $(\bar{x}, \bar{y})$ of the region between the graphs of $y = x^2$ and $x = y^2$. Use symmetry to simplfy your computations.

6. (20 points) Compute

$$\int\int\int_D z^2(x^2 + y^2 + z^2)^{-1/2}\, dV$$

where D is the region between the spheres of radii 2 and 3 centered at the origin.

7. (25 points) Find the area A of the region in the first quadrant bounded by the curves $xy = 1$, $xy = 2$, $y = x$ and $y = 4x$ by making the change of variables $x = u/v$, $y = v$.

8. (20 points) Compute

$$\int_C 2y^3\,dx + (x^4 + 6y^2 x)\,dy$$

where C is the boundary of the region in the first quadrant bounded by $y = 0$, $x = 0$ and $x^4 + y^4 = 1$, oriented counterclockwise.

9. (25 points) Let D be the solid region bounded on top by the plane $z = y$, on the bottom by the plane $z = 0$ and on the sides by the parabolic cylinder $y = x^2$ and the plane $y = x + 2$. Compute

$$\int\int_\Sigma \text{grad} f \cdot \mathbf{n}\,dS$$

where Σ is the boundary of D, $\mathbf{n}$ the outward normal and $f(x, y, z) = x^2 + y^2 - z^2$.