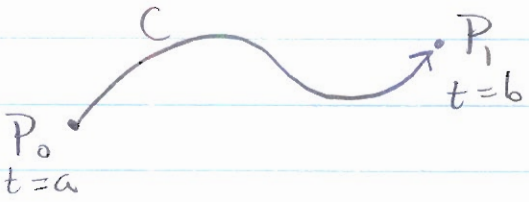
"Magnetic Fields Do No Work!"

Here is an interesting bit of physics that you now have the ability to understand.

Assume there is a charged particle (say, an electron) with charge q moving around in a magnetic field $\vec{B}(x,y,z)$ in vacuum. Assume that its motion from point P_0 to point P_1 can be described by the smooth curve C given by the parameterized curve $\vec{r}(t)$, $a \le t \le b$.



Now, the magnetic field $\vec{B}$ exerts a force on the particle given by the continuous vector field $\vec{F}(x,y,z)$:

$$\vec{F} = q\,\vec{v} \times \vec{B} = q\,\frac{d\vec{r}}{dt} \times \vec{B}$$

where $\vec{v} = \frac{d\vec{r}}{dt}$ is the velocity of the particle at time t. This is called the <u>Lorentz Force Law</u>.

According to Equation 7 of Section 15.2, the work done on the particle by the field $\vec{B}$ is given by

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F} \cdot \frac{d\vec{r}}{dt}\, dt$$

Substituting in the Lorenz force, we see that

$$W = \int_a^b (q\vec{v} \times \vec{B}) \cdot \vec{v}\, dt = q \int_a^b (\vec{v} \times \vec{B}) \cdot \vec{v}\, dt$$

However, $\vec{v} \times \vec{B}$ is some vector that is perpendicular to both $\vec{v}$ and $\vec{B}$, so the dot product of this vector with $\vec{v}$ must be 0.

Thus, $W = 0$, that is, <u>magnetic fields do no work</u>! (The lazy bums...)

This means that magnetic fields can cause a particle to move around in funny ways, but can not speed them up or slow them down.