# Worksheet for Sections 11.1 and 11.2

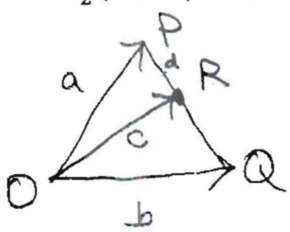
0. (Review) Recall that the line normal to the graph of a function $f$ at $(a, f(a))$ is the line perpendicular to the tangent line at $(a, f(a))$. Find an equation for the line in the $xy$ plane normal to the graph of $y = \sin x$ at $(\pi/4, \sqrt{2}/2)$.

$y = \sin x$ $\quad \frac{dy}{dx} = \cos x$ $\quad \frac{dy}{dx}\Big|_{x=\pi/4} = \cos \pi/4 = \sqrt{2}/2$ = slope of tangent line

Slope of normal line $= -1/$slope of tan line $= -2/\sqrt{2} = -\sqrt{2}$

thus normal line $\quad y - \frac{\sqrt{2}}{2} = -\sqrt{2}\,(x - \pi/4)$

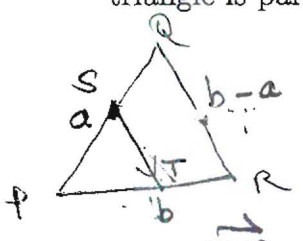
1. Let **a** and **b** be two vectors with initial points at the origin and terminal points $P$ and $Q$ respectively. Show that the vector **c** directed from the origin to the midpoint of $PQ$ is $\frac{1}{2}(\mathbf{a}+\mathbf{b})$. You will need this result to solve the next two problems.




$d = \vec{PR} = \frac{1}{2}\vec{PQ} = \frac{1}{2}(b-a)$

$\therefore C = \vec{OR} = \vec{OP} + \vec{PR} = a + \frac{1}{2}(b-a) = \frac{1}{2}(a+b)$

2. Use vectors to prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.



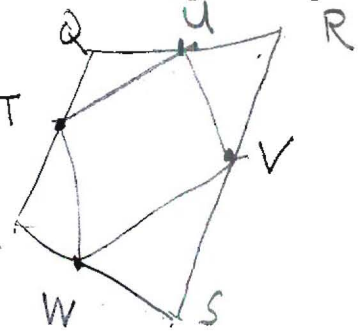

$a = \vec{PQ}$

$b = \vec{PR}$

$\vec{PS} = \frac{a}{2}$ $\quad \vec{PT} = \frac{b}{2}$ $\quad \therefore \vec{PS} + \vec{ST} = \vec{PT}$

$\vec{ST} = \vec{PT} - \vec{PS} = \frac{b}{2} - \frac{a}{2} = \frac{1}{2}(b-a) = \frac{1}{2}\vec{QR}$

~~3. Use vectors to prove that the midpoints of the sides of any quadrilateral are the vertices~~ of a parallelogram.




Let $\vec{PQ} = a$ $\quad \vec{QR} = b$ $\quad \vec{PS} = c$ $\quad \vec{SR} = d$

$\vec{TU} = \frac{1}{2}a + \frac{1}{2}b$ $\quad \vec{WV} = \frac{1}{2}c + \frac{1}{2}d$

But $a + b = c + d$ and so $\boxed{\vec{TU} = \vec{WV}}$

$\vec{TW} = \frac{1}{2}c - \frac{1}{2}a$ $\quad \vec{UV} = \frac{1}{2}b - \frac{1}{2}d$

$\vec{TW} - \vec{UV} = \frac{1}{2}(c+d) - \frac{1}{2}(b+a) = 0.$ $\quad \boxed{\vec{TW} = \vec{UV}}$

This shows WTUV is a Parallelogram