Worksheet for Sections 11.3 and 11.4

0. (Review) Evaluate

$$\int_0^{\pi/4} \frac{\tan x}{\sqrt{\sec x}}\, dx$$

1.

(i) Give a simple example to show that the cross product is not associative, i.e. it is not generally true that

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$$

(ii) Determine which of the following expressions are vectors, which are scalars, and which are undefined

(a) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ (b) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ (c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ (d) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$
(e) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ (f) $(\mathbf{a} \times \mathbf{b})\mathbf{c}$ (g) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$ (h) $\mathbf{a} \times \mathbf{b} \times \mathbf{c} \times \mathbf{d}$
(i) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})\mathbf{d}$ (j) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \times \mathbf{d}$

2.

(a) Prove that two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular if and only if $\|\mathbf{a}\| \leq \|\mathbf{a}+c\mathbf{b}\|$ for every number c. (Hint: $\|\mathbf{a}\|^2 = \mathbf{a} \cdot \mathbf{a}$ and $\|\mathbf{a} + c\mathbf{b}\|^2 = (\mathbf{a} + c\mathbf{b}) \cdot (\mathbf{a} + c\mathbf{b})$.)

(b) Let $c \neq 0$ and assume that the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to one another. Draw a picture of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a}+c\mathbf{b}$. What can you say about the comparative lengths of $\mathbf{a}$ and $\mathbf{a} + c\mathbf{b}$?

(c) Draw nonperpendicular vectors $\mathbf{a}$ and $\mathbf{b}$. Find a nonzero number c such that $\|\mathbf{a}+c\mathbf{b}\| < \|\mathbf{a}\|$ and include $\mathbf{a} + c\mathbf{b}$ in your sketch.

3. Let $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Determine numbers a and b so that $\mathbf{w} - a\mathbf{u} - b\mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.