

Worksheet for Section 12.4

1. Consider the curve C_1 parameterized by

$$\mathbf{r}(t) = \frac{1-t^2}{1+t^2}\mathbf{i} + \frac{2t}{1+t^2}\mathbf{j}.$$

- (a) Show that C_1 is a part of the unit circle centered at the origin, and find $\mathbf{r}(-1)$, $\mathbf{r}(0)$, and $\mathbf{r}(1)$.
- (b) Find $\lim_{t \rightarrow -\infty} \mathbf{r}(t)$ and $\lim_{t \rightarrow \infty} \mathbf{r}(t)$.
- (c) Show that C_1 is the entire unit circle except one point, and find that point.

2. Consider the curve C_2 parameterized by

$$\mathbf{r}(t) = \frac{t^2+1}{t^2-1}\mathbf{i} + \frac{2t}{t^2-1}\mathbf{j}.$$

- (a) Show that C_2 is part of the hyperbola $x^2 - y^2 = 1$.
- (b) Determine which points, if any, of the hyperbola are not on C_2 .

3. Consider the curve C_3 parameterized by

$$\mathbf{r}(t) = \frac{1-t^2}{1+t^2}\mathbf{i} + \frac{t}{1+t^2}\mathbf{j}.$$

Show that C_3 is a portion of an ellipse, and find an equation of the ellipse in the form $x^2/a^2 + y^2/b^2 = 1$.