Worksheet for Section 12.4

1. Consider the curve C_1 parameterized by

$$\mathbf{r}(t) = \frac{1-t^2}{1+t^2}\mathbf{i} + \frac{2t}{1+t^2}\mathbf{j}.$$

(a) Show that C_1 is a part of the unit circle centered at the origin, and find $\mathbf{r}(-1)$, $\mathbf{r}(0)$, and $\mathbf{r}(1)$.
(b) Find $\lim_{t\to-\infty}\mathbf{r}(t)$ and $\lim_{t\to\infty}\mathbf{r}(t)$.
(c) Show that C_1 is the entire unit circle except one point, and find that point.

a) $x = \frac{1-t^2}{1+t^2}$ $y = \frac{2t}{1+t^2}$ $x^2+y^2 = \frac{(1-t^2)^2}{(1+t^2)^2} + \frac{4t^2}{(1+t^2)^2} = \frac{1-2t^2+t^4+4t^2}{1+2t^2+t^4} = \frac{1+2t^2+4t^2}{1+2t^2+4t^2} = 1$

$r(-1) = 0i - j = -j$ $r(0) = i + 0j = i$ $r(1) = 0 + j = j$

b) $\lim_{t\to-\infty} r(t) = -i + 0j = -i$, $\lim_{t\to\infty} r(t) = -i + 0j = -i$

The missing pt is $-i = (-1,0)$ we cannot have $\frac{1-t^2}{1+t^2} = -1$ this would imply $1-t^2 = -1-t^2$, $1 = -1$ ⊗

every other pt on the circle is represented (range of $x(t)$ is $[-1,1]$ Range of y is $[-1,1]$

2. Consider the curve C_2 parameterized by

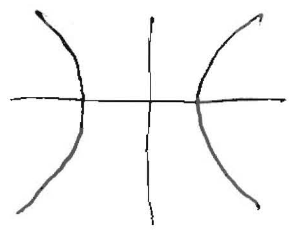
$$\mathbf{r}(t) = \frac{t^2+1}{t^2-1}\mathbf{i} + \frac{2t}{t^2-1}\mathbf{j}.$$

(a) Show that C_2 is part of the hyperbola $x^2 - y^2 = 1$.
(b) Determine which points, if any, of the hyperbola *are not* on C_2.

$x = \frac{t^2+1}{t^2-1}$ $y = \frac{2t}{t^2-1}$ $x^2 - y^2 = \frac{t^4+2t^2+1}{t^4-2t^2+1} - \frac{4t^2}{t^4-2t^2+1} = \frac{t^4-2t^2+1}{t^4-2t^2+1} = 1$

C_2 = whole hyperbola except $(1,0)$ as $x(t) \neq 1$

($\lim_{t\to\infty}(r(t)) = i$ $r(0) = -i = (-1,0)$)



3. Consider the curve C_3 parameterized by

$$\mathbf{r}(t) = \frac{1-t^2}{1+t^2}\mathbf{i} + \frac{t}{1+t^2}\mathbf{j}.$$

Show that C_3 is a portion of an ellipse, and find an equation of the ellipse in the form $x^2/a^2 + y^2/b^2 = 1$.

$x^2 = \frac{1-2t^2+t^4}{1+2t^2+t^4}$ $y^2 = \frac{t^2}{1+2t^2+t^4}$

as in #1 we see that

$x^2 + 4y^2 = 1$

$$\boxed{\frac{x^2}{1} + \frac{y^2}{(1/2)^2} = 1}$$