

## Worksheet for Section 13.6

1. Consider the double cone  $x^2 + y^2 = z^2$ .

- (a) Show that every plane tangent to the double cone passes through the origin.
- (b) Show that every line normal to the double cone passes through the  $z$  axis.

2. Consider the sphere  $x^2 + y^2 + z^2 = a^2$ .

- (a) Let  $P$  be any plane which is tangent to the sphere but not perpendicular to any of the coordinate axes. If the point of tangency is  $(x_0, y_0, z_0)$ , show that the  $x$ ,  $y$ , and  $z$  intercepts of  $P$  are  $a^2/x_0$ ,  $a^2/y_0$ , and  $a^2/z_0$  respectively.
- (b) Show that every line normal to the sphere passes through the origin. Does this happen if the sphere is replaced by a non-spherical ellipsoid? Explain why or why not.

3. Consider the surface

$$x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}.$$

Let  $P$  be any plane which is tangent to the surface and is not parallel to any of the coordinate planes. Show that the sum of the squares of the intercepts of the plane is  $a^2$ .