Solutions 13.6

1 a) $f(x,y,z) = x^2+y^2-z^2 \quad \nabla f = 2x\,i + 2y\,j - 2z\,k$

angent plane at (x_0, y_0, z_0) $\quad 2x_0(x-x_0) + 2y_0(y-y_0) - 2z_0(z-z_0) = 0$

$P: \quad x_0 x + y_0 y + z_0 z = x_0^2 + y_0^2 - z_0^2 = 0$

clearly $(0,0,0)$ is on P.

b) Normal line $\quad x = x_0 + x_0 t \quad y = y_0 + y_0 t \quad z = z_0 - z_0 t$

when $t = -1$ $\quad x = y = 0 \quad z = 2z_0$

$(0, 0, 2z_0)$ is on line

2. $F(x,y,z) = x^2+y^2+z^2 \quad \nabla f = 2x\,i + 2y\,j + 2z\,k$

n. Plane at (x_0, y_0, z_0) $\quad x_0(x-x_0) + y_0(y-y_0) + z_0(z-z_0) = 0$

$x_0 x + y_0 y + z_0 z = x_0^2 + y_0^2 + z_0^2 = a^2$

Intercepts $y = z = 0 \quad x^* = a^2/x_0 \quad x = z = 0 \quad y^* = a^2/y_0 \quad x = y = 0 \quad z^* = a^2/z_0$

b) Normal line $\quad x = x_0 + t x_0 \quad y = y_0 + t y_0 \quad z = z_0 + t z_0$

when $t = -1$ $\quad (x, y, z) = (0, 0, 0)$

This doesn't happen in general $\quad$ Suppose $f = 2x^2 + y^2 + z^2 = 1$

$\nabla f = 4x\,i + 2y\,j + 2z\,k$ $\quad$ Line $\quad x = x_0 + 4x_0 t \quad y = y_0 + 2y_0 t \quad z = z_0 + 2z_0 t$ $\quad$ does not pass thru $(0,0,0)$

c) $f(x,y,z) = x^{2/3} + y^{2/3} + z^{2/3}$

$\nabla f = \frac{2}{3}x^{-1/3}\,i + \frac{2}{3}y^{-1/3}\,j + \frac{2}{3}z^{-1/3}\,k$ if $x \neq 0,\ y \neq 0,\ z \neq 0$.

Tangent Plane at (x_0, y_0, z_0)

$$x_0^{-1/3}(x-x_0) + y_0^{-1/3}(y-y_0) + z_0^{-1/3}(z-z_0) = 0$$

$$x_0^{-1/3}x + y_0^{-1/3}y + z_0^{-1/3}z = x_0^{2/3} + y_0^{2/3} + z_0^{2/3} = a^{2/3}$$

intercepts are $x_0^{1/3}a^{2/3},\ y_0^{1/3}a^{2/3},\ z_0^{1/3}a^{2/3}$

$$(x_0^{1/3}a^{2/3})^2 + (y_0^{1/3}a^{2/3})^2 + (z_0^{1/3}a^{2/3})^2 = (x_0^{2/3} + y_0^{2/3} + z_0^{2/3})a^{4/3} = a^{2/3}a^{4/3} = a^2$$