Worksheet for Sections 13.8-13.9

You will solve the following problem in two ways. Find the point (x, y, z) in the first octant $(x > 0,\ y > 0,\ z > 0)$ for which $3x + 2y + z = 24$ and for which $f(x, y, z) = x^2yz$ is a maximum.

1. Solve the problem by solving the constraint for one of the variables, say z, and making the substitution in the objective function.

$f(x,y,z) = x^2 y z \qquad 3x + 2y + z = 24 \qquad z = 24 - 3x - 2y$

$f(x,y) = x^2y(24 - 3x - 2y) = 24x^2y - 3x^3y - 2x^2y^2$

$f_x = 48xy - 9x^2y - 4xy^2 = 0$ we may divide by $xy > 0$

$f_y = 24x^2 - 3x^3 - 4x^2y = 0$ divide by $x^2 > 0$

$48 - 9x - 4y = 0$ Subtract $24 - 6x = 0$

$24 - 3x - 4y = 0$ $x = 4$

$\cancel{9x}$: $4y = 12$ $y = 3$ $z = 24 - 12 - 6 = 6$

$\underline{\underline{x = 4,\ y = 3,\ z = 6}}$

2. Solve the problem using Lagrange multipliers. Which method is easier?

$\nabla f = 2xyz\,i + x^2z\,j + x^2y\,k$

$\nabla g = 3i + 2j + k$ $\qquad \nabla f = \lambda \nabla g$

$2xyz = 3\lambda$

$x^2z = 2\lambda$

$x^2y = \lambda$

$x^2z = 2x^2y \Rightarrow \underline{\underline{z = 2y}}$

$4xyz = 3x^2z \quad 4y = 3x \quad x = \frac{4}{3}x$

$4y + 2y + 2y = 24 \quad 8y = 24$

$y = 3$

$x = 4$

$z = 6$

as above.