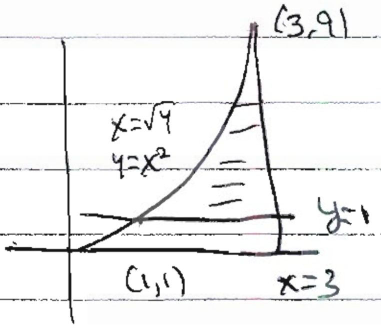
# Worksheet 14.2–14.3 Solutions

1)



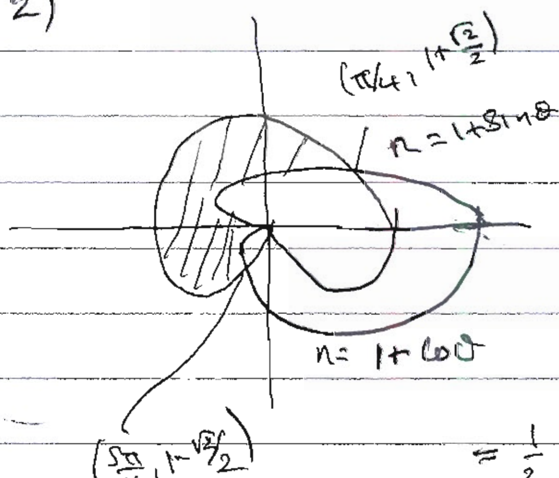

$$\int_1^9\int_{\sqrt{y}}^3 \frac{e^{(x^2-2x)}}{x+1}\,dx\,dy = \int_1^3\int_1^{x^2} \frac{e^{(x^2-2x)}}{x+1}\,dy\,dx$$

$$= \int_1^3 \frac{e^{x^2-2x}}{x+1}(x^2-1)\,dx = \int_1^3 (x-1)e^{x^2-2x}\,dx$$

let $u = x^2-2x$ $\quad$ $x=1$ $u=-1$
$du = (2x-2)\,dx$ $\quad$ $x=3$ $u=3$
$(x-1)dx = \frac{1}{2}du$

$$= \frac{1}{2}\int_{-1}^3 e^u\,du = \frac{1}{2}e^u\Big|_{-1}^3 = \frac{1}{2}(e^3 - e^{-1})$$

2)




$1+\sin\theta = 1+\cos\theta$
$\sin\theta = \cos\theta$
$\theta = \pi/4, 5\pi/4$

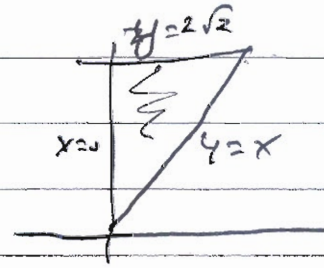
$$A = \int_{\pi/4}^{5\pi/4}\int_{1+\cos\theta}^{1+\sin\theta} r\,dr\,d\theta = \frac{1}{2}\int_{\pi/4}^{5\pi/4}(1+\sin\theta)^2-(1+\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_{\pi/4}^{5\pi/4}(1+2\sin\theta+\sin^2\theta-1-2\cos\theta-\cos^2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{\pi/4}^{5\pi/4}(2\sin\theta-2\cos\theta-\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\left(-2\cos\theta-2\sin\theta-\frac{1}{2}\sin 2\theta\right)\Big|_{\pi/4}^{5\pi/4} = \frac{1}{2}\left[\left(\sqrt{2}+\sqrt{2}-\tfrac{1}{2}\right)-\left(-\sqrt{2}-\sqrt{2}-\tfrac{1}{2}\right)\right] = \frac{1}{2}\cdot 4\sqrt{2} = 2\sqrt{2}$$

3) $f(x,y) = \frac{1}{2}y^2$ $\quad f_x = 0$ $\quad f_y = y$ $\quad dS = \sqrt{1+y^2}\,dy\,dx$




$$S = \int_0^{2\sqrt{2}}\int_0^y \sqrt{1+y^2}\,dx\,dy = \int_0^{2\sqrt{2}} y\sqrt{1+y^2}\,dy$$

$$= \frac{1}{3}(1+y^2)^{3/2}\Big|_0^{2\sqrt{2}} = \frac{1}{3}\left(9^{3/2}-1^{3/2}\right) = \frac{1}{3}(27-1) = 26/3$$