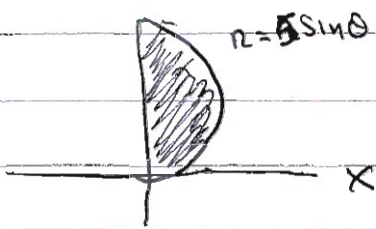


Worksheet 14.5-14.7 Solutions



$$x^2 + y^2 = 5y = r^2 = 5r \sin \theta \quad r = 5 \sin \theta$$

$$V = \int_0^{\pi/2} \int_0^{5 \sin \theta} \int_0^{\sqrt{25-r^2}} r \, dz \, dr \, d\theta$$

(Solved Below)

$$2) \quad S = a - \rho \quad m = \int_0^{2\pi} \int_0^{\pi} \int_0^a (a - \rho) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$= 2\pi \int_0^{\pi} \int_0^a (a\rho^2 - \rho^3) \sin \phi \, d\rho \, d\phi$$

$$= 2\pi \int_0^{\pi} \left(\frac{a\rho^3}{3} - \frac{\rho^4}{4} \right) \Big|_0^a \sin \phi \, d\phi = 2\pi \int_0^{\pi} \left(\frac{a^4}{3} - \frac{a^4}{4} \right) \sin \phi \, d\phi$$

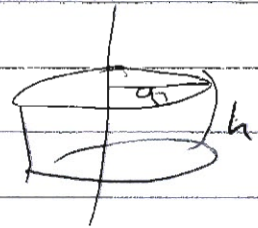
$$= \frac{\pi}{6} a^4 \int_0^{\pi} \sin \phi \, d\phi = \frac{\pi}{6} a^4 (-\cos \phi) \Big|_0^{\pi} = \frac{\pi}{6} a^4 (1 - (-1)) = \frac{\pi a^4}{3}$$

3. $a = \text{radius} = 15 \text{ cm}$
 $h = \text{height} = 3 \text{ cm}$
 $m = \text{mass} = 2 \text{ kg}$

$$V = \pi a^2 h \quad S = \frac{m}{V} = (\pi a^2 h)^{-1} m = \frac{m}{\pi a^2 h}$$

$$I = \int_0^{2\pi} \int_0^a \int_0^h \rho^2 \, dz \, dr \, d\theta = 2\pi \int_0^a \int_0^h \rho^3 \, dz \, dr$$

$$= 2\pi S h \left(\frac{a^4}{4} \right) = \frac{\pi S h a^4}{2}$$



$$I = \frac{\pi}{2} h a^4 \cdot \frac{m}{\pi a^2 h} = \frac{a^2 \cdot m}{2} = (15)^2 = 225$$

Solution of (1) $V = \int_0^{\pi/2} \int_0^{5 \sin \theta} r (25 - r^2)^{1/2} \, dr \, d\theta$

$$= \int_0^{\pi/2} \left[-\frac{(25 - r^2)^{3/2}}{3} \right]_0^{5 \sin \theta} \, d\theta = \frac{1}{3} \int_0^{\pi/2} (25 - 25 \sin^2 \theta)^{3/2} + 25^{3/2} \, d\theta$$

$$= \frac{125}{3} \int_0^{\pi/2} (1 - \cos^2 \theta) \, d\theta = \frac{125}{3} \int_0^{\pi/2} (1 - \cos \theta + \sin^2 \theta \cos \theta) \, d\theta$$

$$\begin{aligned} \cos^3 \theta &= \cos^2 \theta \cos \theta \\ &= (1 - \sin^2 \theta) \cos \theta \end{aligned}$$

$$= \frac{125}{3} \left(\theta - \sin \theta + \frac{\sin^3 \theta}{3} \right) \Big|_0^{\pi/2} = \frac{125}{3} \left[\frac{\pi}{2} - 1 + \frac{1}{3} \right]$$

$$= \frac{125}{3} \left[\frac{\pi}{2} - \frac{2}{3} \right]$$