

## Worksheet for Section 15.3

1. Let  $g$  be a smooth function of one variable, and let

$$\mathbf{F}(x, y, z) = [g(x^2 + y^2 + z^2)](x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \quad (1)$$

(a) Show that  $\mathbf{F}$  is conservative.

(Hint: Show that  $\mathbf{F} = \nabla f$ , where  $f(x, y, z) = \frac{1}{2}h(x^2 + y^2 + z^2)$  and  $h(u) = \int g(u) du$ .)

(b) Show that  $\mathbf{F}$  is irrotational. i.e.  $\nabla \times \mathbf{F} = \mathbf{0}$ .

Since central force fields have the form (1), parts (a) and (b) show that they are both conservative and irrotational.

2. If a charge  $q$  is located at the origin, then according to Coulomb's law the electric force  $\mathbf{E}(x, y, z)$  exerted by the charge on a positive unit charge located at a point  $(x, y, z)$  other than the origin is given by the equation

$$\mathbf{E}(x, y, z) = \frac{q}{4\pi\epsilon_0(x^2 + y^2 + z^2)^{3/2}}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

where  $\epsilon_0$  is a constant called the permittivity of empty space. Assume that an electron is located at the origin and has a charge of  $-1.6 \times 10^{-19}$  coulombs. Find the work  $W$  done by the electric field on a positive unit charge that moves from a distance of  $10^{-11}$  meters to a distance of  $10^{-12}$  meters from the electron. Take  $4\pi\epsilon_0 = 1.113 \times 10^{-10}$ . Your answer will be in joules.

3. Suppose the speed of an object having a mass of 5 kilograms and moving in a conservative force field decreases from 50 meters per second to 10 meters per second. Find the increase in potential energy of the object. (Your answer will be in joules.)