

Worksheet 15.3 Solutions

1. (a) $f(x, y, z) = \frac{1}{2} h(x^2 + y^2 + z^2)$, $h(u) = \int q(u) du$

$$f_x = \frac{1}{2} h'(x^2 + y^2 + z^2) \cdot 2x = x g(x^2 + y^2 + z^2)$$

$$f_y = y g(x^2 + y^2 + z^2) \quad f_z = z g(x^2 + y^2 + z^2) \quad \therefore F = \nabla f$$

b) $\nabla \times F = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xg & yg & zg \end{vmatrix} = (2yzg' - 2yzg')i + (2xzg' - 2zxg')j + (2xyg' - 2xyg')k = \underline{0}$

2. $E = \nabla \phi \quad \phi = \frac{-q}{4\pi\epsilon_0} \frac{1}{r} \quad r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$

$$W = \phi(10^{-12}) - \phi(10^{-11})$$

$$= \frac{-q}{4\pi\epsilon_0} (10^{12} - 10^{11}) = \frac{9(1.6 \cdot 10^{-19})}{1.113 \cdot 10^{-10}} 10^{11} = 10^2 \cdot \frac{9 \cdot 1.6}{1.113} = 1294 \text{ Joules}$$

2. Loss of KE

$$= \frac{1}{2} 5(50)^2 - \frac{1}{2} 5(10)^2$$

$$= 2.5 \cdot 2400 = 6000 \text{ Joules.}$$