Take-home Final exam. Problem 1. Due May 17, 11:59 PM

Consider a spatial competing species model given by the following system of PDEs:

$$u_t = D(x,y)[u_{xx} + u_{yy}] + u(1 - u - v); \quad (1)$$

$$v_t = D(x,y)[v_{xx} + v_{yy}] + 0.75v(1 - (4/3)v - (2/3)u), \quad (2)$$

Here, u and v represent the densities of the species A and B respectively. Suppose their habitat is restricted to a square-shaped national park surrounded with a fence and no-one can escape from it. Assume that the park is a unit square. The diffusion coefficient is nonuniform. It is determined by the landscape of the park and given in the file 'Diff.mat'. To read the data from it, type:

```
data = load('Diff.mat');
D = data.a;
```

Suppose that initially the species A and B are concentrated around the locations $(0.3, 0.7)$ and $(0.7, 0.3)$ respectively (the two spots with relatively low diffusion coefficient). Model the initial distributions using the Gaussian functions

$$u(x,y,0) = \exp(-4((x-0.3)^2 + (y-0.7)^2)); \quad (3)$$

$$v(x,y,0) = \exp(-4((x-0.7)^2 + (y-0.3)^2)). \quad (4)$$

Set up an Initial and Boundary Value Problem (IBVP) for this model. Specify the boundary conditions. Pick any appropriate numerical method and solve the IBVP on the time interval $[0,3]$*. Plot the solutions at* $t = 3$*. Plot the graphs of the averages* `umean` *and* `vmean` *of* u *and* v *with respect to the spatial coordinates as functions of* t*. Estimate the time (using your numerical solution) when* u *and* v *become almost uniform, i.e., when their deviations from* `umean` *and* `vmean` *do not exceed 10%. Predict the behavior of* $u(x,y,t)$ *and* $v(x,y,t)$ *for large values of* t*.*

Submit a SINGLE(!) .m file with your code. Your .m file should include all functions called. Accompany it with a pdf file where you (i) **state the IBVP,** (ii) **specify the numerical method you are using, and** (iii) **answer the question about the long term behavior of** $u(x,y,t)$ **and** $v(x,y,t)$**.**

Take-home Final exam. Problem 2. Due May 18, 11:59 PM

Consider the viscous Burgers equation

$$u_t + [0.5u^2]_x = 0.01u_{xx}, \quad x \in [0,3], \quad t \geq 0, \tag{1}$$

with the periodic boundary conditions and the initial condition

$$u(x,0) = \begin{cases} 0, & x \in [0,1) \cup (2,3], \\ 1, & x \in [1,2]. \end{cases} \tag{2}$$

In addition, consider the corresponding nonviscous Burgers equation

$$u_t + [0.5u^2]_x = 0, \quad x \in [0,3], \quad t \geq 0, \tag{3}$$

with the periodic boundary conditions and the initial condition (2).

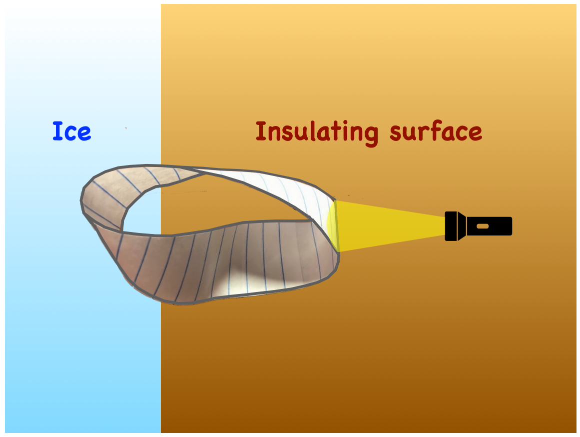
1. *Plot a shock diagram on the xt-plane for Eq. (3) for* $0 \leq t \leq 10$. *Provide analytic formulas for shock lines (curves)* $x^*(t)$ *along which the solution is discontinuous. Plot the shock lines and the characteristics. Also, explain what will be* $u(x,t)$ *as* $t \to \infty$.
2. *Choose an appropriate numerical method and solve Eq. (3) numerically on the interval* $[0,10]$.
3. *Choose an appropriate numerical method and solve Eq. (1) numerically on the time interval* $[0,10]$.
4. *Make your program create a figure displaying the exact solution to Eq. (3) and the numerical solutions to Eqs. (1) and (3) at* $t = 3$, *and similar figures for* $t = 5$ *and* $t = 10$.

Submit a SINGLE(!) .m file with your code. Your .m file should include all functions called. Accompany it with a pdf time with the shock diagram, analytic formulas for the shock lines, and a statement about the behavior of $u(x,t)$ **for Eq. (3) as** $t \to \infty$**. Explain how you have obtained your results.**

Take-home Final exam. Problem 3. Due May 18, 11:59 PM

A Moebius strip is obtained from a rectangle by gluing its ends together with a half twist.

Consider a Moebius strip lying on a flat surface as shown in the schematic figure below.



The Moebius strip touches the ice of temperature zero with a part of its edge of approximately one fourth of its length. The rest of the Moebius strip is lying on an insulating surface. Suppose there is a flash light directed to the strip and heating it as shown in the figure. Assume that the Moebius strip is opaque and made of metal (i.e., it conducts heat quite well). Its size is 1.5 by 4π inches. These settings mean that the heat can come to the strip only due to the light from the flash light and can only escape through the piece of its boundary touching the ice.

Set up a Boundary Value Problem (BVP). You will need to make up a reasonable right-hand side $f(x, y)$ and pick the correct boundary conditions. Convert your BVP to a system of linear algebraic equations $Lu = f$ and solve it using the backslash operator. Your code should be written so that it is easy to change the mesh size. Plot u using the `imagesc` *command.*

Submit a SINGLE(!) .m file with your code. Your .m file should include all functions called.