

MATH 436
HOMEWORK 10
DUE DECEMBER 11, 2007

- (1) Let $\sigma: (0, 2\pi) \times (0, 2\pi) \rightarrow \mathbb{R}^3$ be the surface patch defined by

$$\sigma(u, v) = ((\cos(u) + 3) \cos(v), (\cos(u) + 3) \sin(v), \sin(u)).$$

Compute the Christoffel symbols Γ_{ij}^k for this patch.

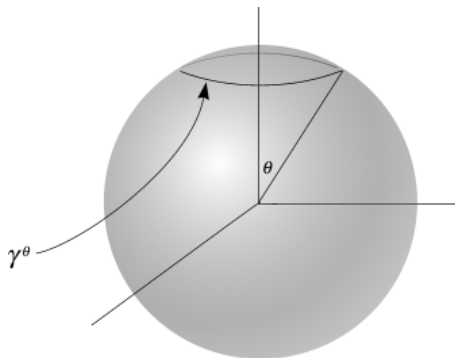
- (2) Let $\sigma: \mathbb{R} \times (0, 2\pi) \rightarrow \mathbb{R}^3$ be the surface patch defined by

$$\sigma(u, v) = (\cosh(u) \cos(v), \cosh(u) \sin(v), u).$$

Compute the Christoffel symbols Γ_{ij}^k for this patch.

- (3) Compute the area of an n -sided polygon, whose sides are all geodesics, on the pseudo-sphere.

- (4) For a fixed $\theta \in (0, \pi)$, let γ^θ be a positively oriented unit-speed parameterization of the circle on S^2 with fixed θ (see figure).



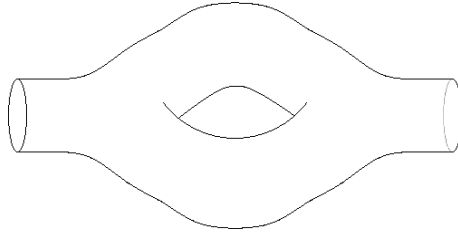
Compute

$$\int_0^{\ell(\gamma^\theta)} \kappa_g ds,$$

i.e. the integral of the geodesic curvature of γ^θ around one period, as a function of θ .

(Turn over the page for the remaining 6 problems).

- (5) A compact “surface with boundary” is the result of removing a finite number of open disjoint disks from a compact surface; for example, the figure below is the result of deleting two disjoint open disks from a torus.



You may assume (the true fact) that the Euler characteristic is well defined for surfaces with boundary. Compute the Euler characteristic of the pictured surface with boundary. If S is a compact surface with boundary with g holes and n disjoint circles for a boundary, guess a formula for the Euler characteristic of S .

OK, I was joking about there being 6 more problems.