MATH 436
HOMEWORK 3
DUE SEPTEMBER 25, 2007

(1) Compute the curvature and torsion of the curve $\boldsymbol{\alpha}\colon \mathbb{R} \to \mathbb{R}^3$ given by

$$\boldsymbol{\alpha}(t) = (a\cosh(t), a\sinh(t), bt)$$

where $a, b \in \mathbb{R}$ are constants.

(2) Let $\boldsymbol{\beta}\colon (-1, 1) \to \mathbb{R}^3$ be defined by

$$\boldsymbol{\beta}(s) = \left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\right).$$

Show $\boldsymbol{\beta}$ is a unit speed curve. Compute its tangent vector, principal normal vector, and binormal vector, and compute each of their first derivatives with respect to s.

(3) Is there a simple closed curve in the plane with length equal to 6 feet bounding an area of 3 square feet? If so, give an example; if not, tell why not.

(4) Let $\boldsymbol{\gamma}\colon \mathbb{R} \to \mathbb{R}^2$ be the curve

$$\boldsymbol{\gamma}(t) = ([\cos(3t) + 2]\cos(t), [\cos(3t) + 2]\sin(t)).$$

Show $\boldsymbol{\gamma}$ is a simple closed curve, and compute the area of its interior.

Date: September 18, 2007.