MATH 436
HOMEWORK 7
DUE NOVEMBER 5, 2007

(1) Compute

$$\int_{S^2} K d\mathcal{A}$$

where S^2 is, as usual, the unit sphere in $\mathbb{R}^3$, and K is the Gaussian curvature. You can choose the atlas (though using angular spherical coordinates lets you get away with only doing one integral - you'll need to say why though).

(2) Suppose a surface S is tangent to a plane P along a curve $\boldsymbol{\gamma}$. Prove that each point on the curve $\boldsymbol{\gamma}$ is either parabolic or planar.

(3) Compute the Gaussian and Mean curvatures of the cylinder

$$\boldsymbol{\sigma}(u, v) = (\cos(u), \sin(u), v).$$

(4) A curve $\mathcal{C}$ on a surface S is a *line of curvature* of S if the tangent vector to $\mathcal{C}$ is a principal vector of S at each point of $\mathcal{C}$. Draw some lines of curvature on
   (a) A sphere
   (b) A torus
   (c) A cylinder

(5) Compute the principal curvatures of the patch

$$\boldsymbol{\sigma}(u, v) = \left(u, v, u^2 - v^2\right)$$

at the point $(0, 0, 0)$.

Date: October 30, 2007.