

MATH 436
HOMEWORK 9
DUE DECEMBER 4, 2007

- (1) Suppose that $\sigma(u, v) = \gamma(u) + v\delta(u)$ is a ruled surface (see Example 4.12 in the text). Show for any fixed u_0 , the curve

$$\alpha(t) = \gamma(u_0) + t\delta(u_0)$$

is a geodesic.

- (2) By a direct computation, show that the equator, parametrized as a unit-speed curve, on the unit sphere S^2 satisfies the geodesic equations, using any parametrization of S^2 you'd like.

- (3) Use the corollary to the Theorema Egregium (Corollary 10.1 in the text) to compute the Gaussian curvature of the patch

$$\sigma(\theta, \varphi) = (\sin(\theta) \cos(\varphi), \sin(\theta) \sin(\varphi), \cos(\theta)).$$

- (4) In the proof of Archimedes's Theorem (see Section 5.5 in the text if you need to refresh your memory) we constructed a conformal map from S^2 to a cylinder. Without computing what this map does to fundamental forms, use the Theorema Egregium to say why the map cannot be an isometry.