

QUIZ
DECEMBER 11, 2007

(All Groups) (a) Find a parameterization for the level curve

$$y^2 - x^2 = 1.$$

(b) Show that $\gamma(t) = \left(\frac{4}{5} \cos(t), 1 - \sin(t), -\frac{3}{5} \cos(t)\right)$ is a unit speed curve. Compute its curvature and torsion.

(c) Compute the first and second fundamental forms of the surface patch

$$\sigma(u, v) = (u - v, u + v, u^2 + v^2).$$

(d) Show if a curve on a surface has zero geodesic and normal curvature everywhere then it is part of a line.

(Group 1) Suppose $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ is a smooth function such that $f_x, f_y,$ and f_z don't all vanish at any point. Let $S = f^{-1}(0)$. Show

$$\nabla f = (f_x, f_y, f_z)$$

is always perpendicular to S , and that S is orientable.

(Group 2) Let $\gamma(s)$ be a unit speed curve in \mathbb{R}^3 , with principal normal \mathbf{n} and binormal \mathbf{b} . The tube of radius a around γ is parameterized by

$$\sigma(s, \theta) = \gamma(s) + a(\mathbf{n}(s) \cos(\theta) + \mathbf{b}(s) \sin(\theta)).$$

Prove that σ is regular if the curvature κ of γ is less than a^{-1} everywhere.

(Group 3) Compute the geodesic curvature of any circle on the unit sphere (not necessarily a great circle).

(Group 4) A surface patch has first fundamental form

$$\cos^2(u)du^2 + \cosh^2(v)dv^2.$$

Compute the Christoffel symbols for this patch.