RESEARCH STATEMENT

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My research falls generally into the area of Topology, and more specifically the study of cell complexes and manifolds. A cell complex is a space X that is built up inductively; for each $n \geq 0$ we have a subspace X_n of X, and $X = \bigcup_n X_n$. The subspace X_0 is a collection of disjoint points (the "0-cells"), and in general X_n is obtained from X_{n-1} by attaching "n-cells" (sometimes called n-disks or n-balls), each of which are homeomorphic to $\{x \in \mathbb{R}^n \mid \|x\| \leq 1\}$. An example of a cell complex is a triangulated surface, like a polyhedron, where the vertices are the 0-cells, the edges are the 1-cells, and the faces are the 2-cells. Surfaces are also examples of two dimensional manifolds, which are spaces which locally look like $\mathbb{R}^2$, but whose global structure is more complex. One can also obtain cell complexes from triangulating higher dimensional manifolds, i.e. spaces which locally look like $\mathbb{R}^n$ for some n.

Cell complexes are good spaces to study, from an algebraic standpoint, because there turns out to be a natural way to associate an algebraic object called a "chain complex" to a cell complex, with which one can perform various algebraic operations to obtain topological invariants of the original space. An example is the Euler characteristic of a finite complex (meaning a cell complex having a finite number of cells), which one obtains by computing the alternating sum of the number of cells in each dimension. This is an extension of the more classical notion of the Euler characteristic of a triangulated surface, $F - E + V$ (the number of faces minus the number of edges plus the number of vertices).

I study an invariant called "Turaev torsion"; [1] and [2] are good general references. The Turaev torsion of a finite complex is only interesting when the Euler characteristic is zero, and in general can be thought of as measuring how complicated the attaching maps are in each dimension, and combining the information from each dimension to get a single invariant. It is related to several other important invariants, especially for 3-manifolds, where there are known relationships to Alexander-Fox invariants, the Seiberg-Witten invariant, and the Casson-Walker-Lescop invariant.

In [2], Turaev proves several results relating the Turaev torsion to cohomology (another invariant of cell complexes) for 3-manifolds without boundary, and asks what analogous results can be obtained for 3-manifolds with boundary. In my thesis, I prove similar formulas to Turaev's, answering his question. I also plan to compare my results to Turaev's by "gluing" together certain manifolds with boundary to obtain manifolds without boundary. In the future, I also would like to compare my results to certain classical results for "link exteriors," which are examples of 3-manifolds with boundary. It is an open problem to relate the Turaev torsion to quantum invariants of 3-manifolds, and I am interested in learning about quantum invariants to study that problem. I also have an interest in other areas of Topology; for example the study of L^2-cohomology, the study of group actions on products of spheres, and Algebraic topology (e.g. the study of Spectra and generalized homology theories).

The computation of torsion often involves some interesting calculations accessible to undergraduates, including the calculation of Alexander invariants from the Wirtinger presentation of the fundamental group of a link exterior. I am also interested in involving undergraduates in geometric visualization problems using technology such as Java and Geomview. My webpage, `http://www.math.umd.edu/~cbtruman/`, includes some Java projects that could be understood and extended by advanced undergraduate students.

References

[1] Vladimir Turaev. *Introduction to combinatorial torsions*. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2001. Notes taken by Felix Schlenk.

[2] Vladimir Turaev. *Torsions of 3-dimensional manifolds*, volume 208 of *Progress in Mathematics*. Birkhäuser Verlag, Basel, 2002.