

Exam 1

Handed out: Wednesday, 09/30/09

READ CAREFULLY. WORK ON ALL QUESTIONS. Justify your answers. Cross out what is not meant to be part of your final answer. You are allowed to use 1 page of handwritten notes of your choice (one sheet of paper with 1 side written only.)

1. Consider the two vectors

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(a)[5 pts] Find the projection of  $\mathbf{b}$  onto  $\mathbf{a}$ .

(b)[5 pts] Find a vector  $\mathbf{c}$  perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ .

2. (a)[5 pts] Find the symmetric equations for the line that passes through the point  $(1, 0, 3)$  and is perpendicular to the plane  $2x - y + z = 8$ .

(b)[5 pts] Find the equation for the plane that contains the point  $(0, 1, 2)$  and is perpendicular to the line with parametric equations  $x = 1 + 2t$ ,  $y = -3 + 5t$ ,  $z = 6t$ .

3. A particle has the position (vector-valued) function

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + \frac{2}{3}t^{3/2}\mathbf{k}, \quad t > 0.$$

(a)[6 pts] Find the velocity, speed, and acceleration of this particle.

(b)[4 pts] Determine the tangential and normal components,  $a_T$  and  $a_N$ , of the acceleration  $\mathbf{a}$  from part (a).

4. (10 pts) Find the length  $L$  of the curve  $C$  parametrized by the vector-valued function

$$\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j} + \frac{3}{2}(\sin^2 t)\mathbf{k}, \quad 0 \leq t \leq \pi/6.$$

**Note:** You may need the integral  $\int \sin x \cos x dx = (1/2) \sin^2 x$