

Exam 2

Handed out: Monday, 10/26/09

READ CAREFULLY. WORK ON ALL QUESTIONS. Justify your answers. Cross out what is not meant to be part of your final answer. You are allowed to use 1 page of your own handwritten notes (1 sheet of paper with 1 side written only).

1. [10 pts] Consider the function

$$f(x, y) = e^{(x+y)^2} + e^{(x-y)^2} .$$

Compute directly the partial derivatives  $\partial f/\partial x$ ,  $\partial f/\partial y$ ,  $\partial^2 f/\partial x^2$ ,  $\partial^2 f/\partial y^2$  for this  $f$ . What is the quantity  $\partial^2 f/\partial x^2 - \partial^2 f/\partial y^2$  equal to ?

2. (a)[5 pts] Consider the function

$$f(x, y, z) = e^x + e^y + e^{2z} .$$

Find the direction in which this  $f$  increases most rapidly at the point  $(1, 1, -1)$ .

What is the maximal directional derivative of  $f$  at the point  $(1, 1, -1)$  ?

(b)[5 pts] Now consider the surface  $S$  described by the equation  $z = g(x, y) = x \sin y$ . Find an equation for the plane tangent to  $S$  at the point  $(x_0, y_0, z_0) = (1, \pi/2, 1)$ .

3. [10 pts] Consider the function

$$w = f(x, y, z) = \frac{x}{y} + \frac{z}{x} ,$$

where  $x = \sin t$ ,  $y = \cos t$ ,  $z = \tan t$ . By use of the Chain Rule, compute the derivative  $dw/dt$ .

**Note:** Your final answer for  $dw/dt$  should depend only on the variable  $t$ . You are not required to simplify the final formula for  $dw/dt$ .

4. [10 pts] The object *distance*  $x$ , image *distance*  $y$ , and focal *length*  $l$  of a simple lens satisfy the relation

$$g(x, y) = \frac{1}{x} + \frac{1}{y} = \frac{1}{l} .$$

Determine the minimum distance  $f(x, y) = x + y$  between the object and the image under the constraint of a given (constant) focal length  $l$ . **Note:** Here, you are asked to find the extreme value of a function under a constraint. Assume that a minimum of  $f$  under the given constraint exists and proceed to find it by a method taught in class.