

Exam 1

Handed out: Monday, 10/01/07

WORK ON ALL PROBLEMS. Justify your answers. Cross out what is not meant to be part of your final answer.

1. Consider the vectors

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}, \quad \mathbf{c} = -5\mathbf{j} + \mathbf{k}.$$

- (a)[5 pts] Show that  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  all lie on the *same* plane.  
(b)[5 pts] Show that  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular and resolve  $\mathbf{c}$  into vectors parallel to  $\mathbf{a}$  and  $\mathbf{b}$ .

**Useful formula:** You can use the formula for the projection of  $\mathbf{c}$  onto  $\mathbf{a}$ :

$$\text{pr}_{\mathbf{a}}\mathbf{c} = \left( \frac{\mathbf{a} \cdot \mathbf{c}}{\|\mathbf{a}\|^2} \right) \mathbf{a}.$$

2. Consider the following vectors:

$$\mathbf{a} = \lambda\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}, \quad \mathbf{c} = \mathbf{i} + \mathbf{j},$$

where  $\lambda$  is an arbitrary real number.

- (a)[7 pts] Find the vector  $\mathbf{d}(\lambda) = \mathbf{a} \times \mathbf{b}$  and the number  $A(\lambda) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$ .  
(b)[3 pts] For what values of  $\lambda$  are  $\mathbf{d} = \mathbf{0}$  and  $A = 0$  simultaneously ?
3. (10 pts) Let  $l$  be the intersection of the planes  $2x - 3y + 4z = 2$  and  $x - z = 1$ . Find an equation for  $l$ .
4. (10 pts) Find the length  $L$  of the curve  $C$  parametrized by the vector

$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \ln t\mathbf{k}, \quad 1 \leq t \leq 2.$$