

Exam 2

Handed out: Friday, 10/26/07

READ CAREFULLY AND WORK ON ALL PROBLEMS. Justify your answers. If you get stuck to a problem, move on to another one. Cross out what is not part of your final answer. Total time: 50min.

1. (10 pts) Consider the function $z = x^{-1}e^{-y/x}$ for $x \neq 0$. Show that z satisfies the equation

$$\frac{\partial z}{\partial x} = y \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} .$$

2. (a)(5 pts) Consider the function $f(x, y) = \sin(xy)$. Find the direction in which f increases most rapidly at the point $(x_0, y_0) = (1, \pi)$. What is the maximal directional derivative of f at this point?

(b)(5 pts) Now consider the function $f(x, y, z) = \sin(xy) + \cos(xz)$. Find an equation for the plane tangent to the level surface $f(x, y, z) = 0$ at the point $(x_0, y_0, z_0) = (\pi/4, 4, 2)$.

3. (10 pts) Consider the function $f(x, y) = -2x^2 + 3xy + y^2 - 4x + 3y - 1$.

(a)(7 pts) Find all critical points of f . Specify which of these points correspond to relative maximum values, relative minimum values and saddle points of f . **Hint:** Apply the second partials test.

(b)(3 pts) Suppose that f takes values in the region R defined by $x^2 + y^2 \leq 4$.

Explain briefly why f has a maximum and a minimum value in R . Are such values attained in the interior or at the boundary of R ? **Hint:** In part (b), you may use your result from (a). Then, the answer to (b) is really short!

4. (10 pts) Consider the function $w = f(y - x, y - z, z - x)$. Assume that partial derivatives of f exist. Compute the sum

$$A = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} .$$