1. [4 pts] Give a formula \( \varphi(x) \) of \( \mathcal{L}_{\bar{N}} \) which defines the set
   \[ X = \{2^0, 2^1, 2^2, \ldots, 2^k, \ldots\} \] in \( \mathcal{N} \). Explain why your formula works.

2. [4 pts] Show that \( X \subseteq \bar{N} \) is definable in \( \mathcal{N} \) iff \( K_X \) is definable in \( \mathcal{N} \).

3. [4 pts] (a) Show that \( \text{PA} \models \forall x(0 \cdot x = 0) \).

   [4 pts] (b) Show that \( \text{PA} \models \forall x \neg(x < x) \).

4. [4 pts] Assume that the binary relation \( R \) is representable in \( \text{PA} \) by the
   formula \( \varphi(x, y) \). Define \( X \) as \( \{k : R(k, l) \text{ holds for some } l\} \). Prove that
   \( k \in X \) iff \( \text{PA} \models \exists y \varphi(x, y) \).

**NOTE:** Your solutions must include enough detail to justify your conclusions.