1. [10 pts] Give an example of a language $L$ (with just finitely many non-logical symbols) and some finite set $\Sigma \subseteq S_{n_L}$ such that the theory $T = Cn(\Sigma)$ has models and all models of $T$ are infinite.

2. [15 pts] Let $L^{nl} = \{R\}$ where $R$ is a binary relation symbol and let $\mathfrak{A} = (\omega, <)$. Let $\mathfrak{B}$ be such that $\mathfrak{A} \equiv \mathfrak{B}$ but $\mathfrak{A}$ is not isomorphic to $\mathfrak{B}$. Prove that there is some infinite sequence $\{b_n\}_{n \in \omega}$ of elements of $B$ which is strictly decreasing, that is $R^B(b_{n+1}, b_n)$ holds for all $n \in \omega$.

3. [15 pts] Let $T_1$ and $T_2$ be theories of $L$, and assume that there is no sentence $\theta$ of $L$ such that $T_1 \models \theta$ and $T_2 \models \neg \theta$. Prove that $(T_1 \cup T_2)$ has a model.

   [Warning: $(T_1 \cup T_2)$ need not be a theory]

4. [15 pts] Let $T_1$ and $T_2$ be theories of $L$. Assume that for every $L$-structure $\mathfrak{A}$ we have

   $\mathfrak{A} \models T_1$ iff $\mathfrak{A} \not\models T_2$.

   Prove that there is some sentence $\sigma$ of $L$ such that $T_1 = Cn(\sigma)$.

NOTE: Your solutions must include enough detail to justify your conclusions.