1. [10 pts] (a) Assume that $\mathfrak{A} \prec \mathfrak{B}$ and $\mathfrak{A} \neq \mathfrak{B}$. Prove that there is no formula $\varphi(x)$ of $L$ such that $\varphi^\mathfrak{B} = \mathfrak{A}$.

[5 pts] (b) Give an example of $\mathfrak{A}$, $\mathfrak{B}$ and $\varphi(x)$ where $\mathfrak{A} \subseteq \mathfrak{B}$, $\mathfrak{A} \neq \mathfrak{B}$, but $\varphi^\mathfrak{B} = \mathfrak{A}$.

2. Let $L^{nl} = \{E\}$ where $E$ is a binary relation symbol. Let $\mathfrak{A}$ be the countable $L$-structure in which $E^\mathfrak{A}$ is an equivalence relation on $A$ with exactly one $E^\mathfrak{A}$ class of size $n$ for every positive integer $n$ but with no infinite $E^\mathfrak{A}$-classes. Define $T = Th(\mathfrak{A})$.

[10 pts] (a) Prove or disprove: $T$ is model complete.

[15 pts] (b) Prove that $\mathfrak{A}$ is a prime model of $T$.

[15 pts] (c) Prove that $T$ is not $\omega$-categorical.

3. [20 pts] Let $T = Th((\mathbb{Z}, <))$. Let $\psi(x, y) = (x < y) \land \neg \exists z(x < z \land z < y)$.

Prove that $\psi(x, y)$ is a complete formula with respect to $T$.

**NOTE:** Your solutions must include enough detail to justify your conclusions.