1. [10 pts] Assume that $X$ and $Y$ are each the range of some recursive function on $\omega$ into $\omega$. Prove that $(X \cap Y)$ is either empty or the range of some recursive function on $\omega$.

2. [10 pts] Let $X \subseteq \omega$. Assume that both $X$ and $(\omega \setminus X)$ are each the range of some recursive function on $\omega$. Prove that $X$ is recursive.

3. [8 pts] Let $R \subseteq \omega \times \omega$ be primitive recursive. Define $F : \omega \times \omega \to \omega$ by $F(k, l) = (\mu n < l)[R(k, n) \text{ holds}]$. Prove that $F$ is primitive recursive.

4. [12 pts] Let $R \subseteq \omega \times \omega$ be recursive. Define $A = \{k \in \omega : R(k, l) \text{ holds for some } l \in \omega\}$.

   (a) Assume that $A$ is non-empty. Prove that $A$ is the image of some recursive function $f : \omega \to \omega$.

   (b) Assume that $A$ is infinite. Prove that $A$ is the image of some 1-1 recursive function $f : \omega \to \omega$.

NOTE: Your solutions must include enough detail to justify your conclusions.