1. [10 pts] Let $E \subseteq \omega \times \omega$ be an equivalence relation on $\omega$. Assume that $E$ is recursive. Prove that there is some recursive function $f : \omega \to \omega$ such that for all $k, l \in \omega$, $f(k) = f(l)$ iff $E(k, l)$ holds.

2. [20 pts] Let $T$ be a consistent theory in a language $L$ such that $\{s, \vec{0}\} \subseteq L^{nl}$. Assume that $T$ has a recursive set of axioms and that $T \models \neg \equiv \vec{n} \vec{m}$ for all $n \neq m$. Prove carefully that every function $f : \omega \to \omega$ which is representable in $T$ is recursive.

3. [20 pts] Assume that $R \subseteq \omega \times \omega$ is r.e. and that $|R_k| = 2$ for every $k \in \omega$. Prove that $R$ is recursive.

4. [25 pts] Assume that $R \subseteq \omega \times \omega$ is r.e., $R_k$ is infinite for all $k \in \omega$, and $(R_k \cap R_l) = \emptyset$ for all $k \neq l$. Prove that there is some recursive $C \subseteq \omega$ such that $|R_k \cap C| = 1$ for all $k \in \omega$.

5. [25 pts] Let $L$ be a language with $L^{nl}$ finite, let $c$ be a constant symbol not in $L$ and let $L' = L \cup \{c\}$. Let $\varphi(x) \in Fm_L$. Define $T' = Cn_{L'}(\varphi(c))$ and $T = Cn_L(\exists x \varphi(x))$. Prove that $T$ is undecidable iff $T'$ is undecidable.

**NOTE:** Your solutions must include enough detail to justify your conclusions.