

Math 130: Midterm 3
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Read carefully the following instructions:

- Write your name, section number, and problem number on EACH of the answer sheets. Do no more than ONE full problem per answer sheet, you may use the back or additional sheets if necessary.
- You may not use any books, notes, or calculators. If your solution contains exponentials or logarithms, you do not need to evaluate them.
- Show all your work and explain everything you write.
- As announced in class and on the webpage, problem Number 0 will be graded on a scale of 0 to 1. This number will multiply the total points for the other problems. To get a grade of 1 you should sketch correctly 3 out of the 4 graphs.
- Exam time: 75 minutes. Solve all 5 problems. The maximum grade is 100.
- Good luck!

Problems:

0. (This problem will be graded on a 0-1 scale. That number will multiply the total points you get for the other problems. To get 1 on this problem you should sketch correctly 3 out of the 4 graphs).

Sketch the graphs of the following functions. Write the values of all intercepts.

(a) $f(x) = e^x$, for $-\infty < x < \infty$

(b) $f(x) = \log_{10} x$, for $0 < x < \infty$

(c) $f(x) = \frac{1}{\log_{10} x}$, for $0 < x < \infty$

(d) $f(x) = \sin(2x)$, for $-\pi \leq x \leq \pi$

1. (30 points)

Let

$$f(x) = xe^{-x^2}$$

It is given that

$$f'(x) = (1 - 2x^2)e^{-x^2}, \quad f''(x) = 2xe^{-x^2}(2x^2 - 3)$$

- (a) (2 point) Find the domain of the function $f(x)$
 - (b) (4 points) Find the x -intercepts and y -intercepts of $f(x)$
 - (c) (2 point) Find the vertical asymptotes of $f(x)$
 - (d) (2 point) Find the horizontal asymptotes of $f(x)$
 - (e) (4 points) Locate all critical points of $f(x)$
 - (f) (4 points) Find all intervals on which $f(x)$ is increasing or decreasing
 - (g) (4 points) Find all intervals on which $f(x)$ is concave downward or concave upward
 - (h) (2 point) Identify the inflection points of $f(x)$
 - (i) (6 points) Graph $f(x)$
2. (a) (10 points) Find $\frac{dy}{dx}$ for the following function

$$y^2x^3 + e^{xy} = 3$$

- (b) (10 points) Find the equation of the tangent line to the curve

$$3x^2 + \ln y = x^3y^2 + 2$$

at the point $(x, y) = (1, 1)$.

3. Let $f(x) = \frac{x}{x^2 + 4}$

- (a) (10 points) Find the relative extrema of $f(x)$.
- (b) (10 points) Find the absolute extrema of $f(x)$ on $[0, 4]$.

4. (30 points)

- (a) (15 points) It is estimated that a person learning a certain assembly-line task takes

$$T(x) = \frac{2 + x}{2 + x^2}$$

minutes to perform the task after x repetitions.

Find $\frac{dT}{dt}$ if $\frac{dx}{dt} = 4$ and 3 repetitions of the task have been performed.

- (b) (15 points) A company wishes to manufacture a box with a volume of 36 ft^3 that is open on top and is twice as long as it is wide. Find the dimensions of the box produced from the minimum amount of material.