

MATH475 MIDTERM ANSWERS.

(1) (a) Nonisomorphic. Upper graph has a vertex of degree 4 and lower graph does not.

(b) Isomorphic. a-2, b-1, c-3, d-5, 3-4, f-7, g-6.

(c) Nonisomorphic. Degree 3 subgraphs are non-isomorphic since upper is connected (bcfe) and lower (2473) is not.

(d) Nonisomorphic. Degree 4 subgraphs are non-isomorphic since lower is connected (45) and lower (ce) is not.

(2) $m = 2$. Indeed $m \geq 1$ since $K_{4,7}$ has more than 2 vertices of odd degree. On the other hand $K_{4,7}$ can be split as an edge disjoint union of two $K_{2,7}$ each of which has an Euler trail.

(3) See next page.

(4) Since Green is the only professor willing to teach MATH310, MATH401 and MATH410 we can not satisfy all the requests since then one of the above course would not be taught. On the other hand the following schedule satisfies 7 requests out of 8:

Brown–240, 246; Green–310, 410; Gray–340, 475; White–140, 401.

(5) The optimal driving time according to the AAA maps is 6:57. For the optimal itinerary see e.g. google maps.

$$P_k(\text{two diamonds connected}) =$$

$$P_k(\text{diamond with diagonal} - \text{diamond}) + P_k(\text{path of 4 nodes} - \text{diamond}) =$$

$$= P_k(\text{diamond with diagonal} - \text{diamond with diagonal}) + 2 P_k(\text{path of 4 nodes} - \text{diamond with diagonal}) +$$

$$P_k(\text{path of 6 nodes}) =$$

$$= k(k-1)^3(k-2)^4 + 2k(k-1)^4(k-2)^2 + k(k-1)^5$$

