# MATH475 SAMPLE EXAMS.

## Exam 1.

(1) How many ways are there to distribute 8 different toys and 8 identical candy to 3 children
(a) without restrictions;
(b) if first child should get exactly 2 toys;
(c) if the first child should get at least 2 toys?

(2) (a) Write a generating function for the number of ways a given number $n$ can be represented as a sum of two odd and three even numbers.
(b) Find a number of ways 50 can be represented as a sum of two odd and three even numbers.

(3) Find a bounded solution to the recurrence relation

$$2a_n = 5a_{n-1} - 2a_{n-2}, \quad a_0 = 1.$$

(4) A bridge hand is called weak in a suit if it has neither ace nor king nor queen in this suit.
(a) Find a number of bridge hands which are weak in at least one suit.
(b) Find a number of bridge hands which are weak in exactly one suit.

## Exam 2.

(1) How many arrangements of the letters PEPPERMILL
(a) do not start and end with the same letter;
(b) have at least one P in the same position as in the original word;
(c) have all Es in the first half?

(2) Find the generating function for a sequence satisfying

$$a_n = 7a_{n-1} - 5a_{n-4} + n, \quad a_0 = 0, \quad a_1 = 1, \quad a_2 = 2, \quad a_3 = 3.$$

(3) Consider the following algorithm for finding two largest among $n$ numbers. Divide the numbers into two equal groups, find the largest in the first ($M_1 > m_1$) and the second ($M_2 > m_2$) group and then find the two largest numbers among $M_1, M_2, m_1, m_2$ by using at most three comparisons ($m_1$ to $M_2$, $m_2$ to $M_1$ and $M_1$ to $M_2$). Find the recurrence relation for the number of comparisons needed in the worst case and solve it for $n = 2^k$.

(4) How many ways are there to put 10 identical red balls and 10 different blue balls into 4 boxes so that there is a box containing
(a) exactly 2 red balls;
(b) exactly 2 blue balls?