AMSC/CMCS 466 – Introduction to Numerical Analysis I

Spring Term 2006

Instructor: Georg Dolzmann

Homework set #10

Problem 1: (a) Write a MATLAB code `trapezoid(f,a,b,n)` to implement the composite trapezoidal rule $T_n(f)$ for a function f on the interval $[a,b]$ with n subintervals (panels). The function f can be written as an inline function or as a separate mfile.

(b) Apply `trapezoid` with $n = 2, 4, 8, 16, 32, 64, 128$ to the following integrals:

$$\frac{2}{\sqrt{\pi}} \int_0^1 e^{-x^2} \, \mathrm{d}x = \mathrm{erf}(1), \qquad \int_0^1 \sqrt{x} \, \mathrm{d}x, \qquad \int_{-4}^4 \frac{\mathrm{d}x}{1+x^2}.$$

(c) Determine the experimental rates of convergence for each of the functions, i.e., suppose that the error is given by Ch^α and identify α. (Recall that the error function erf is a built-in function in MATLAB.) How do your observations compare with the convergence results we proved in class and how important is the regularity of the integrand?

Problem 2: (a) Derive the Newton–Cotes formula on $[a,b] = [0,1]$ with $n = 3$ and nodes $x_0 = 0$, $x_1 = 1/3$, $x_2 = 2/3$, and $x_3 = 1$,

$$N_3(f) = \frac{1}{8}\Big(f(0) + 3f(\tfrac{1}{3}) + 3f(\tfrac{2}{3}) + f(1)\Big).$$

(b) Find the corresponding formula on $[a,b]$ with $h = (b-a)/3$.

(c) Use the error estimate for the Newton–Cotes formulae to show that

$$|N_3(f) - I(f)| \le \frac{3h^5}{80} \max_{a \le x \le b} |f^{(4)}(x)|.$$