AMSC/CMCS 466 – Introduction to Numerical Analysis I

Spring Term 2006

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Homework set #9

Problem 1: (a) Write a MATLAB program `ndd(x,y)` that uses divided differences to find the unique polynomial of degree $\leq n$ in Newton form that interpolates the data points $(x_1, y_1), \ldots, (x_{n+1}, y_{n+1})$. The output should be a vector `c` with the coefficients (the divided differences $f[x_1, \ldots, x_k]$). You can test your program with the data points $x = (3, 1, 5, 6)$ and $y = (1, -3, 2, 4)$ for which we found the divided differences in class.

(b) Write a MATLAB program `neval(c,x,z)` that evaluates the interpolation polynomial using Horner's scheme at a point z. It is convenient to write the vectorized form of the program, i.e., if you use `neval` with a vector of arguments `z` then the result is the vector with the corresponding function values.

(c) Perform an operation count (multiplications/divisions only) for the divided difference algorithm to find the coefficients c in the form we discussed in class (i.e., for a polynomial of degree n with coefficients $c_0, c_1, \ldots, c_n$).

(d) Use the MATLAB codes in (a) and (b) to interpolate the functions

$$f(x) = \exp(2x)\sin(3\pi x), \qquad g(x) = |x|$$

on $[-1, 1]$. Choose $n + 1$ equally spaced points $-1 = x_1 < x_2 < \cdots < x_{n+1} = 1$ with $n = 5$, 10, 15. For each of the functions, make a plot with the interpolation polynomials and the function itself. What can you say about the quality of the approximation and the importance of the differentiability of the functions f and g, respectively?

(e) Repeat (d) with $g(x) = |x|$ and replace the uniform nodes with the Chebyshev nodes. What do you observe?

Problem 2: [Atkinson 3.12] Consider finding a rational function

$$p(x) = (a + bx)/(1 + cx)$$

that satisfies

$$p(x_i) = y_i, \quad i = 1, 2, 3$$

with x_1, x_2, x_3 distinct. Does such a function $p(x)$ exist, or are additional conditions needed to ensure existence and uniqueness of $p(x)$?