AMSC 612 – Numerical Methods for Partial Differential Equations

Spring Term 2004

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Homework set #9

Problem 1: Show that the row-sum norm,

$$\|A\|_\infty = \max_{\alpha=1,\dots,N} \sum_{\beta=1}^{N} |a_{\alpha\beta}|$$

is the operator norm of the $N \times N$ matrix A associated with the $\|\cdot\|_\infty$ norm on $\mathbb{R}^N$, that is,

$$\|A\|_\infty = \max_{u \neq 0} \frac{\|Au\|_\infty}{\|u\|_\infty}.$$

Problem 2: Let $\Omega = [0,1] \times [0,1]$ be the unit square in the plane. Consider the Dirichlet problem

$$-\Delta u = f \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega$$

with

$$f(x,y) = \sin(\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y).$$

Compute the approximate solution by the finite difference discretization based on the five-point-star with $h = 1/5,\ 1/10,\ 1/20,\ 1/40$. Here $\Delta x = \Delta y = h$, i.e., we use the same spacing in the x and in the y direction. What is the experimental rate of convergence in L^∞ and in L^2? You may use that the exact solution is given by

$$u(x,y) = \frac{1}{2\pi^2}\sin(\pi x)\sin(\pi y) + \frac{1}{5\pi^2}\sin(\pi x)\sin(2\pi y).$$

For $J = 5$, use the command `spy` to view and print the structure of the matrix in the linear system. Plot the solution and the error with $J = 20$.