Math 140 - Calculus I - Sections 02**

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Check list for Test 3

Disclaimer: This list of formulae is only intended to assist you in learning the material. You are supposed to master the material in the book, and if a formula in the book is not included in this list, then it does **not** mean that it is not going to be on quizzes, tests, or the final!

Chapter 4

Section 4.1 *Maximum and minimum values.* Definition of maximum and minimum value of a function f. A point c which is an interior point of an interval is a critical point if $f'(c) = 0$ or if f' is not defined.

Maximum-Minimum Theorem: A continuous function has a maximum and a minimum value on a bounded and closed interval $[a, b]$.

The maximum and the minimum value can be attained at critical points or at the end points of the interval.

Section 4.2 *The mean value theorem.* Let f be continuous on $[a, b]$ and differentiable on (a, b). Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(a) - f(b)}{a - b}.$$

Applications: Prove the existence of solutions to equations like $\tan x = 1 - x$ in $(0, 1)$ by applying the mean value theorem to a suitable function (here $f(x) = (x - 1)\sin x$).

Approximation of function values since

$$f(a) - M(b - a) \leq f(b) \leq f(a) + M(b - a)$$

if $|f'(x)| \leq M$ on (a, b).

Section 4.3 *Applications of the mean value theorem.* One of the most important applications is that an antiderivative of a function f can be defined. There are infinitely many antiderivatives for a given function. You should know how to find antiderivatives for elementary functions.

Definition of increasing and decreasing functions, and the corresponding test via the sign of the first derivative of f.

Section 4.4 *Exponential growth and decay.* All processes that are governed by exponential growth or exponential decay follow the simple law $f(t) = f(0)e^{kt}$. You should know how to find the constant k, e.g., if the half-life of a radioactive substance is given.

Section 4.5 *The first and second derivative test.* Definition of relative maximum and minimum values and of relative extrema.

First derivative test: If f' changes from positive to negative at c, then f has a

relative maximum at c.
If f' changes from negative to positive at c, then f has a relative minimum at c.

Second derivative test: Assume that $f'(c) = 0$.
• if $f''(c) < 0$, then $f(c)$ is a relative maximum value of f.
• if $f''(c) > 0$, then $f(c)$ is a relative minimum value of f.
• we cannot conclude if $f''(c) = 0$.

Section 4.6 *Extreme values on arbitrary intervals.* To solve worded problems, read carefully the problem and make a sketch. Label all relevant quantities and find relations between them. Identify the function you want to maximize/minimize and write it as a function of one master variable. Use the techniques in this chapter to find maximum and minimum values. State clearly how you find critical values and how you decide whether they are maximum or minimum values (first and second derivative test).

Section 4.7 *Concavity and inflection points.* Suppose that I is an open interval and that f'' exists.
• If $f'' > 0$ on I, then the graph of f is concave upward (f' is increasing).
• If $f'' < 0$ on I, then the graph of f is concave downward (f' is decreasing).
• An inflection point is a point where the graph of f changes its type of concavity. Note that it is not enough to find the zeros of f'', you must check that the second derivative really changes its sign.

Section 4.8 *Limits at infinity.* Important definitions: finite and infinite limits at $\pm\infty$. Note, e.g., that the limit at ∞ of $f(x)$ is ∞ if for any given N you can find an M such that $f(x) > N$ for $x > M$, i.e., the function f is above any barrier N if x is large enough and the values are above this barrier for all values of x beyond this point. An oscillation function like $x \sin x$ does not have this property!

Section 4.9 *Graphing.* You should know how to sketch the graph of a function by finding x and y intercepts, symmetry properties, relative extrema and intervals on which f is increasing, decreasing, concave upward, concave downward. The knowledge of horizontal and vertical asymptotes is also important.