Math 140 - Calculus I - Sections 02**

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Check list for Test 4

Disclaimer: This list of formulae is only intended to assist you in learning the material. You are supposed to master the material in the book, and if a formula in the book is not included in this list, then it does **not** mean that it is not going to be on quizzes, tests, or the final!

Chapter 5

Section 5.1 *Preparation for the definite integral.* The integral is a limit - the limit of lower sums $L_f(\mathcal{P})$ and of upper sums $U_f(\mathcal{P})$ as the partitions $\mathcal{P}$ get finer and finer. You should know how to find $L_f(\mathcal{P})$ and $U_f(\mathcal{P})$ for a given function.

Section 5.2 *The definite integral.* The definite integral is the unique number I with

$$L_f(\mathcal{P}) \leq I \leq U_f(\mathcal{P})$$

for all partitions $\mathcal{P}$. Notation:

$$I = \int_a^b f(x)\,dx.$$

Riemann sums are a generalization of lower and upper sums. For calculators, the left, right, and midpoint sum are important.

Section 5.3 *Special properties of the definite integral.* We define

$$\int_a^a f(x)\,dx = 0, \qquad \int_b^a f(x)\,dx = -\int_a^b f(x)\,dx \quad \text{for } a < b.$$

For complicated functions, it is useful that

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad \text{if } a < c < b.$$

To estimate values of integrals, the comparison principle is important. If, e.g., $m \leq f(x) \leq M$ on $[a, b]$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

Section 5.4 *The fundamental theorem of calculus.* Differentiation and integration are linked by the following fact: if

$$G(x) = \int_c^x f(t)\,dt$$

then G is differentiable and $G'(x) = f(x)$.

Remember the chain rule! If

$$F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$$

then

$$F'(x) = f(h(x))h'(x) - f(g(x))g'(x).$$

Evaluation of definite integrals by the fundamental theorem:

$$\int_a^b f(x)\,dx = F(b) - F(a) \qquad \text{if } F'(x) = f(x).$$

Section 5.5 *Indefinite integrals and integration rules.* Notation of the indefinite integral:

$$\int f(x)\,dx = F(x) + C.$$

Sometimes the general comparison principle is useful: If $f(x) \le g(x) \le h(x)$ on $[a, b]$, then

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx \le \int_a^b h(x)\,dx.$$

Section 5.6 *Integration by substitution.* Important formula:

$$\int g(f(x))f'(x)\,dx = G(f(x)) + C, \qquad \text{if } G'(x) = g(x).$$

Change of variables:

$$\int_a^b g(f(x))f'(x)\,dx = \int_{f(a)}^{f(b)} g(u)\,du.$$

Important rule: if you define $u = f(x)$ then you have to rewrite the integral in terms of u. Use that

$$du = f'(x)\,dx$$

to rewrite dx in terms of du.

Tip: if you have to find a definite integral for a complicated integrand, then it's sometimes easier to find the indefinite integral and then to evaluate the antiderivative in the limits.

Section 5.7 *The logarithm.* One way to define the logarithm is via a definite integral:

$$\ln x = \int_1^x \frac{1}{t}\,dt.$$

It follows in particular that

$$\int \frac{1}{x}\,dx = \ln|x|, \qquad \int \frac{f'(x)}{f(x)} = \ln|f(x)| + C.$$

Don't forget the absolute value!

Section 5.8 *Another look at area.* The area of a region R in the plane bounded by $f(x)$ and $g(x)$ is given by

$$A = \int_a^b |f(x) - g(x)|\,dx.$$

Don't forget the absolute value - you can only omit it if for example $f(x) \ge g(x)$ for all x in $[a, b]$. Also, sometimes it is easier to write the integral in y instead of x, that is, to view R as bounded by graphs on the y axis rather than on the x axis.