Fall 2007 Math 221 Final Review

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12.4 : Normal Random Variables

1. Let X be a random variable with probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}[\frac{x-\mu}{\sigma}]^2}$$

   Then we call X is a *normal random variable* and f is a normal density function. Also $E(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$. If $\mu = 0$ and $\sigma = 1$ then we say it is *standard normal* and denote the random variable with a Z.

2. Let Z be a standard normal random variable. Then

   - If $z > 0$, $\Pr(Z \leq z) = \Pr(-\infty < Z \leq 0) + \Pr(0 \leq Z \leq z) = .5 + \Pr(0 \leq Z \leq z)$.
   - If $z < 0$, $\Pr(z \leq Z \leq 0) = \Pr(0 \leq Z \leq -z)$.
   - If $z > 0$, $\Pr(Z \geq z) = \Pr(0 \leq Z < \infty) - \Pr(0 \leq Z \leq z) = .5 - \Pr(0 \leq Z \leq z)$.

3. If X is a normal random variable with mean μ and standard deviation σ, then for any a, b,

$$\Pr(a \leq X \leq b) = \Pr(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma})$$

   where Z is the standard normal random variable. You use all of these facts to compute probabilities.

4. **Problems**

   (a) Let Z be the standard normal random variable. Calculate

   - $\Pr(Z \leq 1.4)$.
   - $\Pr(0.02 \leq Z \leq 0.55)$.
   - $\Pr(Z \leq -1.4)$

   (b) (Modified from book problem p 625 # 25) The gestation period of pregnant females of a certain species is normally distributed with a mean of 6 months and a variance of $\frac{1}{4}$. Find the percentage of births that occur after a gestation period of between 6 months and 7 months.

   (c) **Extra problems from textbook**: p 625,626 (15 – 32)

Common mistakes that are easy to avoid

There will be four different T.A.s grading your finals. Different people assign point values differently. It is important that you write your solution and all of your work as correctly as possible!

1. **Units**: Whenever you are solving a word problem, your answer should have units. You should be as specific as possible. For example, if the solution is in "billions of dollars," "billions" is not the correct unit. "Billions" could refer to people, eggs, snowflakes, etc, etc.

2. **Writing down formulas**: Write down every formula that you use. If you do that, but then make some other mistake, you will at least get some partial credit. For example, if you are asked to find the tangent of some angle and you write $\tan(\theta) = \frac{opp}{adj}$, but then miscalculate the length of the opposite side, you will still receive points for the problem.

3. **Rounding**: Do not round unless you are told to do so. You should also realize that a rounded number is not equal to the original number. For example, $\frac{1}{3} \neq 0.333$. It would be correct to write $\frac{1}{3} \approx 0.333$. Simplification and rounding are also not the same. Simplification means that you write the exact number in a nicer form without changing its value.

4. **Change of Limits Rule**: If you are using u-substitution on a definite integral, then you must change the limits of integration. For example, if $u = x^3 + 1$, we have

$$\begin{aligned}
\int_0^2 x^2\sqrt{x^3+1}dx &= \int_1^9 \sqrt{u}\frac{1}{3}du \\
&= \frac{1}{3}\frac{2}{3}u^{\frac{3}{2}}\Big|_1^9 \\
&= \frac{2}{9}(9^{\frac{3}{2}} - 1^{\frac{3}{2}}) \\
&= \frac{2}{9}(27-1) \\
&= \frac{52}{9}
\end{aligned}$$

Notice that the limits of integration changed on the first line. The following is **NOT CORRECT**!!!:

$$\begin{aligned}
\int_0^2 x^2\sqrt{x^3+1}dx &= \int_0^2 \sqrt{u}\frac{1}{3}du \\
&= \frac{1}{3}\frac{2}{3}u^{\frac{3}{2}}\Big|_0^2 \\
&= \frac{1}{3}\frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_0^2 \\
&= \frac{2}{9}(9^{\frac{3}{2}} - 1^{\frac{3}{2}}) \\
&= \frac{2}{9}(27-1) \\
&= \frac{52}{9}
\end{aligned}$$

/\ /\ /\ **NOT CORRECT**

Instead of changing the limits of integration, you can just calculate the indefinite integral first. For example:

$$\begin{aligned}
\int x^2\sqrt{x^3+1}dx &= \int \sqrt{u}\frac{1}{3}du \\
&= \frac{1}{3}\frac{2}{3}u^{\frac{3}{2}}+C \\
&= \frac{2}{9}(x^3+1)^{\frac{3}{2}}+C \\
&\Rightarrow \int_0^2 x^2\sqrt{x^3+1}dx = \frac{1}{3}\frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_0^2 = \frac{52}{9}
\end{aligned}$$

5. **+C**: Don't forget the $+C$ when you are calculating indefinite integrals. This is especially imporatant when you are solving differential equations.

6. **Evaluate basic trig angles**: You should always evaluate basic trig angles; that is, those of the form $k\pi$, $\frac{k\pi}{2}$, $\frac{k\pi}{3}$, $\frac{k\pi}{4}$ and $\frac{k\pi}{6}$ for some integer k. For example, if you get $\sin(\frac{\pi}{6}$, you should then write $\frac{1}{2}$.

7. **Initial value problem**: If you are instructed to write an initial value problem, then you should write both the initial value and the differential equation and box them both.

8. **"but do not solve it"**: Please read the problems carefully. You will waste precious exam time if you attempt to solve a problem that you are only supposed to set up.

9. $+\ldots$: A Taylor series or geometric series should always include a $+\ldots$ at the end. Otherwise it is just a Taylor polynomial or a geometric sum.

10. **Dropping the** lim **too soon**: Until you actually take the limit (for an improper integral), you should write $lim_{b\to\infty}$ (or $lim_{b\to-\infty}$) before every single step.