HW3 Project Problem A for Math 420

(A). In class we discussed the logistic population model with harvesting, which led to the initial-value problem

$$\frac{dP}{dt} = r\,P\left(1 - \frac{P}{k}\right) - h\,, \qquad P(0) = P_0\,.$$

Solutions of this model depend upon the initial value P_0 and the three positive constant parameters $r, k,$ and h. When you use the explicit (forward) Euler method with uniform time-steps of size δ to numerically approximate the solution $P(t)$ of this initial-value problem, you find that $P(n\delta) \approx \pi_n$ where the π_n are computed using the difference equation

$$\pi_{n+1} = \pi_n + \delta \cdot \left[r\,\pi_n \left(1 - \frac{\pi_n}{k}\right) - h \right], \qquad \pi_0 = P_0\,. \tag{1}$$

Here $n = 0, 1, 2, \cdots$ counts the number of time-steps. The idea is to now consider this numerical approximation of the original continuous-time model to be a discrete-time analog of the original model and to study it in its own right. This project will study this discrete-step *logistic mapping model with harvesting* for time-step $\delta = .02$.

(i) Starting with parameters $P_0 = 5000,\ r = 0.11,\ k = 9000,\ h = 240$, characterize the behavior of the trajectories of π_n. By varying each of these parameters singly, leaving the others fixed, find the value of the parameter being varied at which the population collapses.

(ii) Now look at the trajectories if each of the parameters varies over time periodically around their fixed values, with amplitudes $A_r,\ A_k,\ A_h$ which may be up to $0.2r, 0.2k,$ and $0.2h$ respectively, with periods which may be either the same or different. (Try taking these periods to be of the order of 30–80 time-steps.)

(iii) Now explore what happens if each of $r, k,$ and h are replaced at each time-step n by independent random numbers $u_n \sim \text{Unif}(r - A_r, r + A_r)$, $v_n \sim \text{Unif}(k - A_k, k + A_k)$, $w_n \sim \text{Unif}(h - A_h, h + A_h)$, where the amplitudes $A_r,\ A_r,\ A_h$ are as in (ii), and where $\text{Unif}(a, b)$ denotes (independent) uniform random numbers within the interval (a, b).

In (ii) and (iii), are there amplitudes at which the behavior of the trajectories changes qualitatively?

(iv) Explore these non-constant parameter variations in any combinations you choose. Can you reach any conclusions about which kinds of parameter changes lead to qualitative behavioral changes of the fisheries model (1) with the initially given set of parameters and initial condition?

(v) Can you find other somewhat different combinations of initial conditions and parameters for which smaller-amplitude perturbations can have qualitatively catastrophic effects?