Numerical simulation of twin jets of matter emerging from an active galactic nucleus using the equations of special relativistic hydrodynamics

Darran Furnival

September 30, 2002

Introduction

- Observations show that radio galaxies at the center of galactic clusters inject jets of matter into the surrounding ICM at relativistic speeds.

- This can be modelled by considering a fluid in some region of space bound gravitationally by the surrounding dark matter and inserting jets via a source term or boundary conditions

- It is the purpose of this project to carry out such simulations using the equations of special relativistic hydrodynamics.

Scientific Justification

- It will contribute towards knowledge of black-holes and their impact on the surrounding environment.

- The impact of the jets has an affect on the properties of the cluster which in turn affect cosmological arguments, e.g. the age and curvature of the universe.

- It will assist in explaining discrepancies between model predictions and observational data, e.g. cooling flow models and data gathered by *XMM Newton*.

- We will be able to discern certain properties about the cluster, e.g. its age, from comparing observations with numerical simulations.

Equations of Special Relativistic Hydrodynamics

- Equations in *conservation law* form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}^i(\mathbf{U})}{\partial x^i} = 0$$

$$\mathbf{U} = \begin{bmatrix} D \\ S^1 \\ S^2 \\ S^3 \\ \tau \end{bmatrix} \quad \mathbf{F}^i = \begin{bmatrix} Dv^i \\ S^1 v^i + p\delta^{1i} \\ S^2 v^i + p\delta^{2i} \\ S^3 v^i + p\delta^{3i} \\ S^i - Dv^i \end{bmatrix}$$

$D \equiv$ rest mass density

$S^i \equiv$ momentum density

$\tau \equiv$ energy density

- Conserved variables related to primitive variables by:

$$
\begin{aligned}
D &= \rho\Gamma \\
S^i &= \rho h \Gamma^2 v^i \\
\tau &= \rho h \Gamma^2 - p - D \\
h &= 1 + \epsilon + \frac{p}{\rho}
\end{aligned}
$$

$$
\begin{aligned}
\rho &\equiv \text{proper rest mass density} \\
h &\equiv \text{specific enthalpy} \\
v^i &\equiv \text{velocity} \\
p &\equiv \text{pressure} \\
\epsilon &\equiv \text{internal energy}
\end{aligned}
$$

$\Gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor.

- Equation of state:

$$p = p(\rho, \epsilon)$$

- Also need to include gravitational potential due to dark matter.

Algorithms

- Use Cartesian coordinate system.

- Use method of lines, i.e. discretize in space to leave a system of ODEs.
  - Discretize in space using *finite differences*.
  - Use *Riemann solver* to calculate fluxes across fluid elements.
  - Use implicit multistep method, e.g. RK4, to solve ODEs.

- Use *Newton-Raphson* iteration to recover primitive variables.

- Use *domain decomposition* to optimize for parallel processors.

Software Verification

- Check code against standard problems, e.g. Riemann problem, for which the exact solution is known.

- Certain physical quantities are conserved, e.g. mass-energy, momentum, etc. Discritization causes loss of conservation which should diminish as grid becomes finer. Code can be checked against such convergence theorems.

Language, Platform and Visualization

- The code will be written in Fortran 90 as a matter of personal preference.

- The Beowulf cluster in the astrophysics department will be the platform that the code will ultimately be designed to run on as this is the platform that is most readily accessible for those involved in the project and those most likely to use the completed software.

- The visualization will most likely be done in IDL as this is used extensively in the astrophysics department and has the ability to make movies.