Math 406 – Fall 2009 – Harry Tamvakis

PROBLEM SET 2 – Due September 17, 2009

Reading for this week: Sections 1 and 2.

Problems

From the textbook: Section 1, Problems #2, 4, 6, 11, Section 2, Problems #2, 3, 5, 10. In addition, do the following problems:

A1) a) In 1511, Carolus Bouvellus claimed that for $n \geqslant 1$ one or both of $6n-1$ and $6n+1$ were prime. Show that this conjecture is false.

b) Bouvellus must have realized something was amiss because he soon revised his claim to read that every prime, except 2 and 3, can be expressed in the form $6n \pm 1$, for some natural number n. Show that this conjecture is true.

c) Prove that $\{3, 5, 7\}$ is the only set of three consecutive odd numbers that are all prime.

A2) a) How many natural numbers less than or equal to 1000 are divisible by 3? By 5? By 7?

b) How many natural numbers less than or equal to 1000 are divisible by 3 or by 5?

c) How many natural numbers less than or equal to 1000 are divisible by 3, 5, or 7?

A3) Prove that if $n > 4$ is composite, then n divides $(n-1)!$. Conversely, show that if n is prime, then n does *not* divide $(n-1)!$.

A4) Find all prime numbers p such that $17p + 1$ is a perfect square.

Extra Credit Problems.

EC1) Determine, with proof, all prime numbers p such that $p+10$ and $p+20$ are also prime.

EC2) Let m and n be positive integers and suppose that a is an integer greater than 1. Use the Euclidean algorithm to prove that

$$\gcd(a^m - 1, a^n - 1) = a^{\gcd(m,n)} - 1.$$