PROBLEM SET 4 – Due September 28, 2023

Reading for this week: Sections 3 and 4 (except for Theorem 6 on page 31).

Problems

From the textbook: Section 3, Problems #2, 4, 5, 6; Section 4, Problems #2, 3, 4, 6, 19. In addition, do the following problems:

A1) Let \( \tau(n) \) be the number of positive divisors of \( n \). Show that \( \tau(n) = \tau(n + 1) = \tau(n + 2) = \tau(n + 3) \) if \( n = 3655 \).

A2) An old receipt has faded. It reads 88 chickens at a total of \$x4.2y \), where \( x \) and \( y \) are unreadable digits. How much did each chicken cost?

A3) (a) Prove that the integer \( 111^{333} + 333^{111} \) is divisible by 7.
(b) Prove that the integer \( 53^{103} + 103^{53} \) is divisible by 39.

Extra Credit Problems.

EC1) Use congruences to find the last two digits of \( 2^{100} \).

EC2) A positive integer is called polite if it can be represented as a sum of two or more consecutive positive integers. For example, 7 and 22 are polite since \( 7 = 3 + 4 \) and \( 22 = 4 + 5 + 6 + 7 \), while 2 is impolite. Prove that the only impolite positive integers are the powers of 2, that is, 1, 2, 4, 8, 16, \ldots