Math 406 – Fall 2009 – Harry Tamvakis
PROBLEM SET 4 – Due October 1, 2009

Reading for this week: Section 4 and whatever we cover from Section 5.

Problems

From the textbook: Section 4, Problems #10, 12, 14, 18, 20, Section 5, #2, 6, 20. From the Additional Problems for Section 4, do #4.4 and 4.6 (on page 184) and #4.10 (on page 185). In addition, do the following problems:

A1) An old receipt has faded. It reads 88 chickens at a total of \$ $x4.2y$, where x and y are unreadable digits. How much did each chicken cost?

A2) Solve the following linear congruences:

$$\text{a) } 16x \equiv 27 \mod 29, \quad \text{b) } 20x \equiv 16 \mod 64,$$
$$\text{c) } 22x \equiv 5 \mod 12, \quad \text{d) } 131x \equiv 21 \mod 77.$$

A3) a) Find the smallest positive integer n such that $n+1$, $n+2$, $n+3$, and $n+4$ are all composite.

b) If k is any positive integer, prove that the number $k!+1$ is followed by $k-1$ consecutive composite integers. (Note: $k! = 1 \cdot 2 \cdot 3 \cdots k$.)

A4) Show that if a and b are integers and $a+b$ is even, then 24 divides $ab(a^2-b^2)$.

Extra Credit Problems.

EC1) a) Find a seven-digit number with all its digits different, which is divisible by each of its digits.

b) Does there exist an eight-digit number with the same property? Justify your answer.

EC2) The number 1 is written on a blackboard. After each second the number on the blackboard is increased by the sum of its digits, producing a sequence of numbers $1, 2, 4, 8, 16, 23, 28, 38, 49, 62, \ldots$ Will the number 123456 ever be written on the blackboard?