Math 406 – Fall 2009 – Harry Tamvakis
PROBLEM SET 5 – Due October 15, 2009

Reading for this week: Sections 5 and 6.

Problems

From the textbook: Section 5, Problems #4, 12, 17, Section 6, #2, 4, 15, 17, 18. In addition, do the following problems:

A1) From Fermat's theorem deduce that 13 divides $11^{12n+6}+1$ for all nonnegative integers n.

A2) a) Find the remainder when 15! is divided by 17.

b) Find the remainder when 2(26!) is divided by 29.

A3) If p and q are distinct prime numbers, prove that

$$p^{q-1}+q^{p-1} \equiv 1 \pmod{pq}.$$

A4) A deck of cards is shuffled by cutting the deck into two piles of 26 cards. Then, the new deck is formed by alternating cards from the two piles, starting with the bottom pile.

a) Show that if a card begins in the k-th position in the deck, it will be in the d-th position in the new deck, where $d \equiv 2k \mod 53$ and $1 \leqslant d \leqslant 52$.

b) Determine the number of shuffles of the type described above that are needed to return the deck of cards to its original order.

Extra Credit Problems.

EC1) From Section 6, problem #20.

EC2) A *complete system of residues modulo* n is a set of n numbers such that no two of them are congruent modulo n. Let p be an odd prime and let $a_1, \ldots, a_p$ and $b_1, \ldots b_p$ be complete systems of residues modulo p. Prove that $a_1b_1, \ldots, a_pb_p$ is not a complete system of residues modulo p.