Math 406 – Fall 2009 – Harry Tamvakis
PROBLEM SET 6 – Due October 22, 2009

Reading for this week: Sections 7, 8, and 9.

Problems

From the textbook: Section 7, Problems #12, 16, Section 8, #2, 5, 7, 13, Section 9, #2, 14, 18. In addition, do the following problems:

A1) Prove that n is a perfect number if and only if $\displaystyle\sum_{d|n} \frac{1}{d} = 2$. For example, 6 is perfect because

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2.$$

A2) Find the last two digits in the decimal expansions of 3^{1000} and 7^{999999}.

A3) Prove that $2^{15} - 2^3$ divides $a^{15} - a^3$ for any integer a. [Hint: $2^{15} - 2^3 = 5 \cdot 7 \cdot 8 \cdot 9 \cdot 13$.]

Extra Credit Problems.

EC1) (a) Consider the number $m = 111 \cdots 1$ with n digits, all ones. Prove that if m is prime, then n is prime.

(b) Is the converse of the statement in (a) true?

EC2) Consider the canonical factorization of 1000! into prime powers:

$$1000! = 2^a 3^b 5^c 7^d 11^e \cdots$$

Compute the exponents a, b, c, and d.