Math 406 – Spring 2009 – Harry Tamvakis
PROBLEM SET 9 – Due November 12, 2009

Reading for this week: Section 11.

Problems

From the textbook: Section 11, #2, 4, 6, 8, 10, 11, 12, 16, 20. In addition, do the following problems.

A1) Let p be an odd prime and $(a, p) = 1$. Show that the quadratic congruence $ax^2 + bx + c \equiv 0 \pmod{p}$ has a solution in x if and only if $b^2 - 4ac$ is either zero or a quadratic residue of p.

A2) If $p = 2^k + 1$ is prime, show that every quadratic nonresidue of p is a primitive root of p. [Hint: Apply Euler's Criterion.]

A3) a) If p is an odd prime and $(ab, p) = 1$, prove that at least one of a, b, or ab is a quadratic residue of p.

b) Given a prime p, show that p divides $(n^2 - 2)(n^2 - 3)(n^2 - 6)$ for some choice of natural number n.

Extra Credit Problem.

EC) The integers a, b, c, d, e, and f satisfy the equation

$$a^2 + b^2 + c^2 + d^2 + e^2 = f^2.$$

Prove that at least two of the numbers a, b, c, d, e, f must be even.