

**MIDTERM EXAM**

- 1) Find all integer solutions to the equation  $40x + 64y = 56$ .
- 2) Find all natural numbers  $n > 1$  such that (i)  $n$  is the square of an integer and (ii)  $1453 \equiv 2713 \pmod{n}$ .
- 3) Consider the number  $n = 23x45678y$ , where  $x$  and  $y$  are unknown digits. We know that  $n$  is a multiple of 15. Find all such numbers  $n$ .
- 4) Use the Euclidean algorithm to compute  $\gcd(77777777, 77777)$ .
- 5) Find a natural number  $n$  such that (i) 210 divides  $n$ , and (ii)  $n$  has exactly 210 positive divisors.
- 6) Suppose that  $a$  and  $b$  are integers such that  $(a, b) = 1$ . If  $c$  divides  $a + b$ , prove that  $(a, c) = 1$ .
- 7) The three most recent appearances of Halley's comet were in the years 1835, 1910, and 1986, while the next appearance will be in 2061. Prove that

$$1835^{1910} + 1986^{2061} \equiv 0 \pmod{7}.$$

- 8) For any two integers  $a$  and  $b$ , prove that  $35 \mid ab(a^{12} - b^{12})$ . [Hint:  $35 = 5 \cdot 7$ ]

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**Extra Credit Problem:**

- EC)** Compute the product of all the positive divisors of  $10!$  .

Problems 1-8 and EC are worth 12/12/14/12/12/12/14/12/15 points, respectively.

**GOOD LUCK!**