1. The position at time $t$ of a particle moving in space is given by the vector

$$r(t) = e^{2t} \mathbf{i} + t \mathbf{j} + 2e^t \mathbf{k}, \quad -1 \leq t \leq 1$$

a) [10] Find the velocity and acceleration as a function of time.

b) [5] Find the speed as a function of time.

c) [10] Find the tangential and normal components of acceleration $a_T$ and $a_N$.

d) [10] Find $T$ and the curvature $\kappa$ as functions of time.

e) [5] Find the length of the curve parameterized by $r$.

2. Suppose the curve $C_1$ is parameterized by $r_1(t) = t \mathbf{i} - t^2 \mathbf{j} + t^3 \mathbf{k}$ and the curve $C_2$ is parameterized by $r_2(t) = (1 + t) \mathbf{i} + (t - 1) \mathbf{j} + \mathbf{k}$.

a) [5] For which curve is the unit normal $N$ not defined? Why?

b) [10] Find a parametric equation of the line tangent to the curve $C_1$ at the point $(1, -1, 1)$.

c) [10] Find an equation of the plane tangent to both curves $C_1$ and $C_2$ at the point $(1, -1, 1)$.

3.

a) [10] Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} - \mathbf{k}$. You may leave your answer in terms of inverse trigonometric functions.

b) [10] If $\mathbf{F}$ and $\mathbf{G}$ are differentiable vector valued functions, write down the formulae for the derivatives $(\mathbf{F} + \mathbf{G})'$, $(\mathbf{F} \times \mathbf{G})'$, and $(\mathbf{F} \cdot \mathbf{G})'$.

4. [15] Indicate all correct answers for each question (There may be more than one correct answer or no correct answer.)

i) The dot product of two perpendicular vectors is:
   a) a vector. b) a scalar. c) always zero.

ii) The cross product of two perpendicular vectors is:
   a) a vector. b) a scalar. c) always zero.

iii) If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors parallel to the same plane then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ is:
   a) a vector. b) a scalar. c) always zero.

iv) If a particle travels with constant speed, its velocity $\mathbf{v}(t)$ is:
   a) always zero. b) tangent to the curve it travels. c) perpendicular to the curve it travels.

v) If you travel twice as fast around a curve, the curvature will:
   a) increase. b) decrease. c) remain the same.