1. (30) Let \( S = \text{Span}( (1, 0, 1, 0)^T, (0, 1, 0, 1)^T, (1, 2, 3, 4)^T ) \).
   a) Find an orthogonal basis for \( S \).
   b) Find a basis for \( S^\perp \).

2. (20) Let \( u_1, u_2, u_3 \) be an orthonormal set in an inner product space \( V \).
   a) Calculate \( ||u_1 - 2u_2 + 4u_3|| \).
   b) Show that \( u_1 - 2u_2 \) is orthogonal to \( 2u_1 + u_2 \).
   c) Are \( u_1, u_2, u_3 \) linearly independent?
   d) Is \( u_1, u_2, u_3 \) a basis for \( V \)?

3. (25) Let \( L: P_3 \to P_4 \) be the map \( L(p) = xp(x) \).
   a) Show that \( L \) is a linear transformation.
   b) Find the Kernel and Range of \( L \).
   c) Find the matrix of \( L \) with respect to the bases \([x - 1, 1, x^2]\) of \( P_3 \) and \([1, x, x^2, x^3]\) of \( P_4 \).

4. (20) Show that if \( A \) is an \( m \times n \) matrix and \( x \) is in \( N(A^T A) \), then \( x \) is in \( N(A) \).

5. (30) True (always true), False (always false), Maybe (sometimes true and sometimes false, depending on \( A \), \( S \), etc.) or short answer. \( A \) is a \( 4 \times 8 \) matrix, and \( S \) is a subspace of a seven dimensional inner product space \( V \). Also \( A \) has rank 3 and \( S \) has dimension 3.
   a) The null space \( N(A) \) has dimension 1.
   b) If \( x \perp y \) and \( y \perp z \) then \( x \perp z \).
   c) Let \( x_1 \) and \( x_2 \) be two different least squares solutions to \( Ax = b \). Then \( Ax_1 = Ax_2 \).
   d) If \([u_1, u_2, u_3]\) is a basis for \( S \), and \( x \perp u_i \) for each \( i \), then \( x \) is in \( S^\perp \).
   e) If \( AB = 0 \) then the column space of \( B \) is contained in \( N(A) \).
   f) If \( B \) and \( C \) are similar matrices then \( \det(B) = \det(C) \).
   g) \( \dim S^\perp = \) ___.
   h) \( \dim N(A^T) = \) ___.
   i) Give an example of an inner product on \( C[0, 2] \).
   j) If \([u_1, u_2, u_3]\) is an orthogonal basis for \( S \), what is the formula for the projection of \( x \) to \( S \)?